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A VARIATIONAL METHOD OF LEADINGOUT OF EQUATIONS OF MOTION BOUNDARY TERMS IN A ROUND PLATE

For the obtain of equations of equilibrium and boundary terms in a round plate, variation principle of Lagrange is used for complete energy of the stress system. The got equalizations fully coincide after a form with the proper terms and equalizations for the plates of classic theory. A difference is brought in only by coefficients which take into account a transverce shear and compression. These equations take into account inertia of rotation of cross section of plate and influence of normal stress additionally.

Keywords: *transtropic plates, equilibrium of thin plates, transverce shear, compression, equations of equilibrium.*

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ВАРІАЦІЙНИЙ МЕТОД ВИВЕДЕННЯ РІВНЯНЬ РУХУ ТА ГРАНИЧНИХ УМОВ У КРУГЛІЙ ПЛАСТИНІ

Для виведення рівнянь рівноваги та граничних умов у круглій плиті, використано варіаційний принцип Лагранжа для повної енергії пружної системи. Отримані рівняння цілком співпадають за формою з відповідними умовами та рівняннями для пластин класичної теорії. Відмінність вносять лише коефіцієнти, що враховують поперечний зсув та обтиснення. Дані рівняння враховують додатково інерцію обертання поперечних перерізів пластини та вплив нормального напруження.

Ключові слова: *транстропні пластини, рівновага тонких пластин, поперечний зсув, обтиснення, рівняння рівноваги.*

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ВАРИАЦИОННЫЙ МЕТОД ВЫВОДА УРАВНЕНИЙ ДВИЖЕНИЯ И ПРЕДЕЛЬНЫХ УСЛОВИЙ В КРУГЛОЙ ПЛАСТИНЕ

Для получения уравнений равновесия и граничных условий в круглой пластине используется вариационный принцип Лагранжа для общей энергии упругой системы. Полученные уравнения вполне согласуются по форме с соответствующими условиями и уравнениями для пластин классической теории. Разница производится только коэффициентами, которые учитывают поперечный сдвиг и обжимку. Эти уравнения учитывают дополнительную инерцию вращения поперечных сечений пластины и влияние нормального напряжения.

Ключевые слова: *транстропные пластины, равновесие тонких пластин, поперечный сдвиг, обжимка, уравнения равновесия.*

Raising of a problem. There are different methods of outputing of differential equations of equilibrium of thin plates. One of the basic and most widespread methods of outputing of such equations is the use of equations of equilibrium in moments and efforts. The lack of this method is unmotivated recording of boundary terms on the edges of plate. More perfect are variation methods (Lagrange, Reyssner, mixed method) are power explained, which allow, together with equations of equilibrium, to destroy boundary terms. In many articles of domestic and foreign researchers use calculation equations of plates and shells in the nonclassical raising [1, 6]. The most existent nonclassical theories of plates and shells take into account deformation of transverse displacement, and some, partly, take into account and transversal compression. However, as researches show, in calculations for the actions of the contact and local loadings, it should be absolutely taken into account transversal compression, that allows to satisfy most terms on-the-spot contact of plate with other bodies or bases [6].

A research purpose is a leadingout of equations of motion of transtropic plates of medium thickness, which take into account both the effects of transverse shear and deformation of transverse compression, transverse normal tension and inertia of rotation of cross sections.

For the leadingout of equations of motion and boundary terms in a round plate, a will take the a dvantage of variation principle of Lagrange for complete energy of the elastic system [2, 4]

$$\delta \Pi = \delta A , \tag{1}$$

where

$\delta\Pi = \iiint\limits_{V_p} \left(\sigma_r \cdot \delta\varepsilon_r + \sigma_\theta \cdot \delta\varepsilon_\theta + \sigma_z \cdot \delta\varepsilon_z + \tau_{r\theta} \cdot \delta\gamma_{r\theta} + \tau_{rz} \cdot \delta\gamma_{rz} + \tau_{\theta z} \cdot \delta\gamma_{\theta z} \right) dV_p$ – variation of potential energy of deformation; $dV_p = rdrd\theta dz$ it is a volume of plate;

$\delta A = \iiint\limits_{V_p} \left(F_r \delta U + F_\theta \delta V + F_z \delta W \right) dV_p + \iint\limits_S \left(q^- \delta W^- - q^+ \delta W^+ \right) dS$ – variation of work of volume and superficial forces; $dS = rd\theta dr$ - it is an element of surface of a plate; $W^\pm$ -are components of the vector of elastic displacement moving on the external surfaces $z = \pm h$ of plate $F_r = -\rho \dfrac{\partial^2 U}{\partial t^2}$; $F_\theta = -\rho \dfrac{\partial^2 V}{\partial t^2}$; $F_z = -\rho \dfrac{\partial^2 W}{\partial t^2}$ - are projections of forces of inertia on the proper co-ordinate axes, attributed to unit of volume, which carry out the role of volume forces legalistically; ρ – it is a closeness.

Using the formulas of Koshi for component of deformation

$$\varepsilon_r = \frac{\partial U}{\partial r},\ \varepsilon_\theta = \frac{U}{r} + \frac{1}{r}\frac{\partial V}{\partial \theta},\ \varepsilon_z = \frac{\partial W}{\partial z},$$

$$\gamma_{r\theta} = \frac{1}{r}\frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial r} - \frac{V}{r},\ \gamma_{rz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial r},\ \gamma_{z\theta} = \frac{\partial V}{\partial z} + \frac{1}{r}\frac{\partial W}{\partial \theta},$$

but carrying out a varying taking into account the formulas of integration parts and of correlations of type $\delta\left(\dfrac{\partial U}{\partial r}\right) = \dfrac{\partial}{\partial r}(\delta U)$, variation of potential energy will be

$$\delta\Pi = \iiint\limits_{V_p} \left[\frac{\partial}{\partial r}(\sigma_r \delta U) + \frac{\partial}{\partial \theta}\left(\frac{\sigma_\theta}{r}\delta V\right) + \frac{\partial}{\partial z}(\sigma_z \delta W) + \frac{\partial}{\partial \theta}\left(\frac{\tau_{r\theta}}{r}\delta U\right) + \frac{\partial}{\partial r}(\tau_{r\theta}\delta V) + \right.$$
$$\left. + \frac{\partial}{\partial z}(\tau_{rz}\delta U) + \frac{\partial}{\partial r}(\tau_{rz}\delta W) + \frac{\partial}{\partial z}(\tau_{z\theta}\delta V) + \frac{\partial}{\partial \theta}\left(\frac{\tau_{\theta z}}{r}\delta W\right) \right] dV_p - \iiint\limits_{V_p} \left(\frac{\tau_{r\theta}}{r}\delta V - \frac{\sigma_\theta}{r}\delta U + \right.$$
$$+ \frac{\partial \sigma_r}{\partial r}\delta U + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta}\delta V + + \frac{\partial \sigma_z}{\partial z}\delta W + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}\delta U + \frac{\partial \tau_{r\theta}}{\partial r}\delta V + \frac{\partial \tau_{rz}}{\partial z}\delta U + \frac{\partial \tau_{rz}}{\partial r}\delta W +$$
$$\left. + \frac{\partial \tau_{\theta z}}{\partial z}\delta V + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}\delta W \right) dV_p = \iint\limits_{S_r} r\sigma_r \delta U d\theta dz + \iint\limits_{S_\theta} \sigma_\theta \delta V drdz + \iint\limits_{S_z} r\sigma_z \delta W drd\theta + \qquad (2)$$
$$+ \iint\limits_{S_\theta} \tau_{r\theta}\delta U drdz + \iint\limits_{S_r} r\tau_{r\theta}\delta V d\theta dz + \iint\limits_{S_z} r\tau_{rz}\delta U drd\theta + \iint\limits_{S_r} r\tau_{rz}\delta W d\theta dz + \iint\limits_{S_z} r\tau_{z\theta}\delta V drd\theta +$$
$$+ \iint\limits_{S_\theta} \tau_{z\theta}\delta W drdz - \iiint\limits_{V} \left(\frac{\sigma_r}{r}\delta U + \frac{\tau_{r\theta}}{r}\delta V + \frac{\tau_{rz}}{r}\delta W + \frac{\tau_{r\theta}}{r}\delta V - \frac{\sigma_\theta}{r}\delta U + \frac{\partial \sigma_r}{\partial r}\delta U + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta}\delta V + \right.$$
$$\left. + \frac{\partial \sigma_z}{\partial z}\delta W + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}\delta U + \frac{\partial \tau_{r\theta}}{\partial r}\delta V + \frac{\partial \tau_{rz}}{\partial z}\delta U + \frac{\partial \tau_{rz}}{\partial r}\delta W + \frac{\partial \tau_{\theta z}}{\partial z}\delta V + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}\delta W \right) dV_p .$$

Variation of potential of external forces

$$\delta A = -\rho \iiint\limits_{V_p} \left(\frac{\partial^2 U}{\partial t^2}\delta U + \frac{\partial^2 V}{\partial t^2}\delta V + \frac{\partial^2 W}{\partial t^2}\delta W \right) dV_p + \iint\limits_S q_2 \delta\tilde{W} ds . \qquad (3)$$

If to put to equality (2) and (3) in variation equalization (1) and to equate with a zero in a triple integral expressions near independent variations, δU, δV, δW, then will be got differential equalizations of motion of voxel of plate in cylindrical coordinate system:

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}++\frac{\partial\tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=\rho\frac{\partial^2 U}{\partial t^2},$$
$$\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{\theta z}}{\partial z}+\frac{2\tau_{r\theta}}{r}=\rho\frac{\partial^2 V}{\partial t^2}, \qquad (4)$$
$$\frac{\partial\tau_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta}+\frac{\partial\sigma_z}{\partial z}+\frac{\tau_{rz}}{r}=\rho\frac{\partial^2 W}{\partial t^2}.$$

In an order to get equation of equilibrium through forses and moments, and also boundary terms on the edges of a plate, will take expressions for the stresses of presentation through internal forses and moments:

$$\sigma_r=\frac{N_r}{2h}+\frac{M_r z}{I}+\frac{(1-\alpha)\tilde E}{G'}f_0(z)\left(\frac{\partial Q_r}{\partial r}+\frac{\nu}{r}\frac{\partial Q_\theta}{\partial\theta}+\frac{\nu}{r}Q_r\right)+$$
$$+\frac{(1-\alpha)\tilde E h^2}{2E'}f_0(z)\left(\frac{\partial^2 q_2}{\partial r^2}+\frac{\nu}{r^2}\frac{\partial^2 q_2}{\partial\theta^2}+\frac{\nu}{r}\frac{\partial q_2}{\partial r}\right)+A'\big(q_1+q_2\left(f_0(z)-1\right)\big)+zA'\rho\frac{\partial^2 W}{\partial t^2};$$
$$\sigma_\theta=\frac{N_\theta}{2h}+\frac{M_\theta z}{I}+\frac{(1-\alpha)\tilde E}{G'}f_0(z)\left(\nu\frac{\partial Q_r}{\partial r}+\frac{1}{r}\frac{\partial Q_\theta}{\partial\theta}+\frac{1}{r}Q_r\right)+$$
$$+\frac{(1-\alpha)\tilde E h^2}{2E'}f_0(z)\left(\nu\frac{\partial^2 q_2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 q_2}{\partial\theta^2}+\frac{1}{r}\frac{\partial q_2}{\partial r}\right)+A'\big(q_1+q_2\left(f_0(z)-1\right)\big)+zA'\rho\frac{\partial^2 W}{\partial t^2};$$
$$\tau_{r\theta}=\frac{N_{r\theta}}{2h}+\frac{H_{r\theta} z}{I}+\frac{(1-\alpha)\tilde E}{G'}f_0(z)\left(\frac{\partial Q_\theta}{\partial r}+\frac{1}{r}\frac{\partial Q_r}{\partial\theta}+\frac{1}{r}Q_\theta\right)+$$
$$+\frac{(1-\alpha)\tilde E h^2}{2E'}f_0(z)\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial q_2}{\partial\theta}+\frac{\partial q_2}{\partial r}\right); \qquad (5)$$
$$\tau_{rz}=\frac{G'}{K'}\left(1-\frac{z^2}{h^2}\right)Q_r;\ \ \tau_{\theta z}=\frac{G'}{K'}\left(1-\frac{z^2}{h^2}\right)Q_\theta;$$

where $f_0(z)=\frac{z}{4h^3}\left(0,6h^2-z^2\right)$.

Write down expressions for component of the elastic displacement, in the following kind:

$$U(r,\theta,z)=u+\gamma_r z+\psi_r\left(\frac{z}{5}-\frac{z^3}{3h^2}\right);$$
$$V(r,\theta,z)=v+\gamma_\theta z+\psi_\theta\left(\frac{z}{5}-\frac{z^3}{3h^2}\right); \qquad (5à)$$
$$W(r,\theta,z)=w+\frac{2\alpha_0}{E'}q_1 z+\frac{3\alpha_0}{4hE'}\tilde q_2 z^2\left(1-\frac{z^2}{6h^2}\right)-A'\left(z\theta_0+\frac{z^2\theta_1}{2}-\frac{z^4\theta_3}{4h^2}\right)=$$
$$=w+\frac{2\alpha_0}{E'}q_1 z+\frac{3\alpha_0}{4hE'}\tilde q_2 z^2\left(1-\frac{z^2}{6h^2}\right)-A'z\theta_0+\frac{1}{2}A'z^2\theta_3\left(3h^2+\frac{z^2}{2h^2}\right)+\frac{1}{2}A'z^2\Delta w,$$

where γ_r, γ_θ – are the generalized corners of turn; ψ_r, ψ_θ – are functions of transversal change.

Using formulas (5), formulas for deformations and expressions for component of the elastic displacement (5a), find expression for variation of potential energy:

$$\delta\Pi = \iint_S \int_{-h}^{h} \left(\sigma_r \cdot \delta\varepsilon_r + \sigma_\theta \cdot \delta\varepsilon_\theta + \sigma_z \cdot \delta\varepsilon_z + \tau_{r\theta} \cdot \delta\gamma_{r\theta} + \tau_{rz} \cdot \delta\gamma_{rz} + \tau_{\theta z} \cdot \delta\gamma_{\theta z}\right) dz dS =$$

$$= \iint_S \left(N_r \delta\left(\frac{\partial u}{\partial r}\right) + M_r \delta\left(\frac{\partial \gamma_r}{\partial r}\right) + \frac{N_\theta}{r}\delta u + \frac{M_\theta}{r}\delta\gamma_r + N_\theta \delta\left(\frac{\partial v}{r\partial\theta}\right) + M_\theta \delta\left(\frac{\partial\gamma_\theta}{r\partial\theta}\right) + \right.$$

$$+ N_{r\theta}\delta\left(\frac{\partial u}{r\partial\theta}\right) + H_{r\theta}\delta\left(\frac{\partial\gamma_r}{r\partial\theta}\right) + N_{r\theta}\delta\left(\frac{\partial v}{\partial r}\right) + H_{r\theta}\delta\left(\frac{\partial\gamma_\theta}{\partial r}\right) - \frac{N_{r\theta}}{r}\delta v - \frac{H_{r\theta}}{r}\delta\gamma_\theta +$$

$$\left. + Q_r\delta\gamma_r + Q_r\delta\left(\frac{\partial\tilde{w}}{\partial r}\right) + Q_\theta\delta\gamma_\theta + Q_\theta\delta\left(\frac{\partial\tilde{w}}{r\partial\theta}\right)\right) dS .$$

Using the formulas of integration of part and varying on independent variables $u, v, \tilde{w}, \gamma_r, \gamma_\theta$, will get for $\delta\Pi$ and δA:

$$\delta\Pi = -\iint_S \left[\left(\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial\theta} + \frac{N_r - N_\theta}{r} \right)\delta u + \left(\frac{\partial N_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_\theta}{\partial\theta} + \frac{2N_{r\theta}}{r} \right)\delta v + \right.$$

$$+ \left(\frac{\partial Q_r}{\partial r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial\theta} + \frac{Q_r}{r} + q_2 \right)\delta\tilde{w} + \left(\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial H_{\theta r}}{\partial\theta} + \frac{M_r - M_\theta}{r} - Q_r \right)\delta\gamma_r + \qquad (6)$$

$$\left. + \left(\frac{\partial H_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial\theta} + \frac{2}{r}H_{r\theta} - Q_\theta \right)\delta\gamma_\theta \right] dS + \int_L \left[\left(N_r l + N_{r\theta} m\right)\delta u + \left(N_{r\theta} l + N_\theta m\right)\delta v + \right.$$

$$\left. + \left(Q_r l + Q_\theta m\right)\delta\tilde{w} + \left(M_r l + H_{r\theta} m\right)\delta\gamma_r + \left(H_{r\theta} l + M_\theta m\right)\delta\gamma_\theta \right] dL .$$

$$\delta A = -2\rho h \iint_S \left(\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{h^2}{3}\frac{\partial^2\gamma_r}{\partial t^2}\delta\gamma_r + \frac{h^2}{3}\frac{\partial^2\gamma_\theta}{\partial t^2}\delta\gamma_\theta \right) dS + \iint_S \tilde{q}_2 \delta\tilde{w} dS.$$

Here L – is a limit of the contour of area, S, l and m - are direction cosines of normal to the contour of plate.

If to equate expressions near independent variations in (6), obsessed system of equations of motion through internal forces and moments:

$$\begin{aligned}
&\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial\theta} + \frac{N_r - N_\theta}{r} = 2\rho h\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial N_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_\theta}{\partial\theta} + \frac{2N_{r\theta}}{r} = 2\rho h\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial H_{\theta r}}{\partial\theta} + \frac{M_r - M_\theta}{r} - Q_r = \frac{2}{3}\rho h^3\frac{\partial^2\gamma_r}{\partial t^2},\\
&\frac{\partial H_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial\theta} + \frac{2}{r}H_{r\theta} - Q_\theta = \frac{2}{3}\rho h^3\frac{\partial^2\gamma_\theta}{\partial t^2},\\
&\frac{\partial Q_r}{\partial r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial\theta} + \frac{Q_r}{r} = -q_2 + 2\rho h\frac{\partial^2\tilde{w}}{\partial t^2},
\end{aligned} \qquad (7)$$

where $\tilde{w} = w + \frac{1}{6}A'\Delta w h^2 + \frac{9\alpha_0 h A_2 q_2}{40E'}$; $\left\{N_r, N_\theta, N_{r\theta}, Q_r, Q_\theta\right\} = \int_{-h}^{h}\left\{\sigma_r, \sigma_\theta, \tau_{r\theta}, \tau_{rz}, \tau_{\theta z}\right\}dz$;

$\left\{M_r, M_\theta, H_{r\theta}\right\} = \int_{-h}^{h}\left\{\sigma_r, \sigma_\theta, \tau_{r\theta}\right\}zdz$; $Q_r = K'\cdot\psi_r$; $Q_\theta = K'\cdot\psi_\theta$; q^+, q^- – it is loading on the external surfaces of plate ($z = \pm h$), which are directed downward, in direction of axis Oz; G' – it is the module of transversal shear of material of plate; ψ_r, ψ_θ – are deformations of transversal shear of middle surface of a plate.

Equation (7) can be got from the system (4), if, inheriting S.O.Ambartsumyan [1], to increase all equalizations of the system (4) on dz, and first two yet and on zdz and to integrate them in limits from $-h$ to h. Together with that, such method does not allow to get power correct boundary terms on verge of plate.

Boundary terms will get from contour to the integral which is included in equation (6):

$$\left(N_r l + N_{r\theta} m\right)\delta u = 0; \quad \left(N_{r\theta} l + N_\theta m\right)\delta v = 0; \quad \left(Q_r l + Q_\theta m\right)\delta w = 0;$$

$$\left(M_r l + H_{r\theta} m\right)\delta\gamma_r = 0; \quad \left(H_{r\theta} l + M_\theta m\right)\delta\gamma_\theta = 0. \tag{8}$$

From the system of equations (8) it is possible to get static or geometrical boundary terms, depending on what multiplier to equate with a zero.

Putting an equation (4) in place of internal forces and moments their expressions, taking into account previous remarks, will get equalization of motion through moving u, v, w_τ and corners of turn γ_r, γ_θ :

$$\begin{aligned}
&\Delta u + \frac{1+\nu}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial v}{\partial r} - \frac{1}{r}\frac{\partial u}{\partial\theta} + \frac{v}{r}\right) - \frac{1}{r^2}\left(u + 2\frac{\partial v}{\partial\theta}\right) = -\frac{\nu''(1+\nu)}{E}\frac{\partial q_1}{\partial r} + 2\rho h\frac{\partial^2 u}{\partial t^2};\\
&\Delta v + \frac{1+\nu}{1-\nu}\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{u}{r}\right) - \frac{1}{r^2}\left(v - 2\frac{\partial u}{\partial\theta}\right) = -\frac{2\nu''(1+\nu)}{E(1-\nu)}\frac{1}{r}\frac{\partial q_1}{\partial\theta} + 2\rho h\frac{\partial^2 v}{\partial t^2};\\
&\Delta\gamma_r + \frac{1+\nu}{2r}\frac{\partial}{\partial\theta}\left(\frac{\partial\gamma_\theta}{\partial r} - \frac{1}{r}\frac{\partial\gamma_r}{\partial\theta} + \frac{\gamma_\theta}{r}\right) - \frac{1}{r^2}\left(\gamma_r + 2\frac{\partial\gamma_\theta}{\partial\theta}\right) =\\
&= \frac{4\psi_r}{5\varepsilon_\tau} - \frac{3}{5}\frac{\nu''(1+\nu)}{hE}\frac{\partial q_2}{\partial r} + \frac{\rho}{\tilde{E}}\frac{\partial^2\gamma_r}{\partial t^2} - A'\frac{\rho}{\tilde{E}}\frac{\partial^3\widehat{W}}{\partial r\partial t^2};\\
&\Delta\gamma_\theta + \frac{1+\nu}{1-\nu}\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial\gamma_r}{\partial r} + \frac{1}{r}\frac{\partial\gamma_\theta}{\partial\theta} + \frac{\gamma_r}{r}\right) - \frac{1}{r^2}\left(\gamma_\theta - 2\frac{\partial\gamma_r}{\partial\theta}\right) =\\
&= \frac{8\psi_\theta}{5\varepsilon_\tau(1-\nu)} - \frac{6}{5}\frac{\nu''(1+\nu)}{(1-\nu)hE'}\frac{1}{r}\frac{\partial q_2}{\partial r} + \frac{\rho}{\tilde{E}}\frac{\partial^2\gamma_\theta}{\partial t^2} - A'\frac{\rho}{r\tilde{E}}\frac{\partial^3\widehat{W}}{\partial\theta\partial t^2};\\
&K'\Delta w_\tau = -q_2 + 2\rho h\frac{\partial^2\tilde{w}}{\partial t^2}.
\end{aligned} \tag{9}$$

Here $\psi_r = \frac{\partial w_\tau}{\partial r} - \frac{1}{r}\frac{\partial\Omega}{\partial\theta}$; $\psi_\theta = \frac{1}{r}\frac{\partial w_\tau}{\partial\theta} + \frac{\partial\Omega}{\partial r}$; $\gamma_r = -\frac{\partial\overline{w}}{\partial r} - \frac{4}{5}\frac{1}{r}\frac{\partial\Omega}{\partial\theta}$; $\gamma_\theta = -\frac{1}{r}\frac{\partial\overline{w}}{\partial\theta} + \frac{4}{5}\frac{\partial\Omega}{\partial r}$;

$\overline{w} = w - \frac{2.4+\chi_0}{3+\chi_0}w_\tau + \frac{q_2 h}{E_0}$; $E_0 = \frac{40}{9}\left(3+\chi_0\right)E'$; $\chi_0 = \frac{3\nu''}{2G/G' - \nu''}$;

$\widehat{W} = w + 0.3A'h^2\Delta w + 0.43\alpha_0 A_2 q_2 h/E'$; $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$.

Thus we got the system of five equations of motion in the sought after functions $u, v, w_\tau, \gamma_r, \gamma_\theta, \Omega$. To these equations maximum terms (8) and initial conditions must be added at $t = 0$:

$$\begin{aligned}
w &= w_0(r,\theta), \quad \frac{\partial w}{\partial t} = w_1(r,\theta),\\
u &= u_0(r,\theta), \quad \frac{\partial u}{\partial t} = u_1(r,\theta),\\
v &= v_0(r,\theta), \quad \frac{\partial v}{\partial t} = v_1(r,\theta),
\end{aligned} \tag{10}$$

where $w_0, v_0, u_0, u_1, v_1, w_1$ – are given components of the initial dicplacement and initial velocity from a point (r,θ).

System of equations of equilibrium (9) in that part, where they describe the bend of plate, it is possible to erect to more usual kind, if in place of sizes $\gamma_r, \gamma_\theta, \psi_r, \psi_\theta$ to put their expressions through functions w, w_τ, Ω :

$$D\Delta^2 w_q = \left(1-\varepsilon_1\Delta+\frac{\varepsilon'\rho h^4}{4G}\frac{\partial^2}{\partial t^2}\right)q_2 - m\left(1-\varepsilon_1 h^2\Delta\right)\frac{\partial^2\tilde{w}(r,t)}{\partial t^2} - m\varepsilon'\frac{\rho h^2}{4G}\frac{\partial^4\tilde{w}}{\partial t^4}; \qquad (11)$$

$$K'\Delta w_\tau = -q_2 + m\frac{\partial^2\tilde{w}}{\partial t^2}; \quad \Delta\Omega - k^2\Omega = \frac{\rho}{G}\frac{\partial^2\Omega}{\partial t^2},$$

where $\varepsilon_1 = \frac{h^2}{10(1-\nu)}\left(8\frac{G}{G'}-3\nu''\right)$; $D=\frac{2}{3}\frac{Eh^3}{1-\nu^2}$; $m=2\rho h$ –is mass of unit of surface of plate;

$\varepsilon' = 0.1\left(8\frac{G}{G'}+\nu''\right)$; $w_q = w+\varepsilon_2 q_2 / D$; $k^2 = \frac{5}{2}\frac{G'}{G}h^{-2}$;

$$\varepsilon_2 = \frac{h^4}{20\left(1-\nu^2\right)}(1-\alpha)\left(\frac{E}{E'}+A'\frac{E}{G'}\right); \ \alpha = \frac{\nu''\cdot G'}{2G}.$$

The got equations (11) take into account inertia of rotation of cross sections of plate and effect of normal tension additionally σ_z . If we set the parameters ε', A' and $\Delta' \equiv 1-\varepsilon\Delta$, to zero, then these factors in these equations (11) will not be taken into account. The unaccount of inertia conduces to the loss of right part in equalization of Helmholtz.

Conclusions. For the obtain of equations of equilibrium and boundary terms in a round plate, variation principle of Lagrange is used for complete energy of the stress system. The got equalizations fully coincide after a form with the proper terms and equalizations for the plates of classic theory. A difference is brought in only by coefficients which take into account a transverce shear and compression. These equations take into account inertia of rotation of cross section of plate and influence of normal stress additionally.

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