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MODELING OF DYNAMIC STRESS STATE FOAM MEDIA UNDER THE ACTION OF CONCENTRATED LOAD

The influence of impulse concentrated load on the distribution of normalized dynamic hoop stresses in the foam medium is investigated in this paper. For the solving of the non-stationary problem in the case of plane deformation for the structurally inhomogeneous materials, the model of Cosserat continuum is applied. This model enables accounting for the influence of shear-rotation deformation of micro-particles of the medium. In the framework of Cosserat elasticity with the applying of the Fourier transforms for time variable and developing of the boundary integral equation method the solving of the non-stationary problem reduces to the system of singular integral equations, where the components that determining the influence of shear-rotation deformations are selected. The numerical calculations were performed for the polyurethane closed-cell foam with tunnel cavity of circular cross-section under the action of localized impulse load. The developed approach can be used to predict the mechanical behaviour of foam materials under the action of time-variable load based on the analysis of the distribution of dynamic stresses.

Keywords: *Impulse Load, Non-stationary Plane Problem, Cosserat Elasticity.*

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МОДЕЛЮВАННЯ ДИНАМІЧНОГО НАПРУЖЕННОГО СТАНУ ПІНИСТИХ МАТЕРІАЛІВ ЗА ДІЇ КОНЦЕНТРОВАНОГО НАВАНТАЖЕННЯ

У роботі досліджується вплив імпульсного зосередженого навантаження на розподіл нормованих динамічних кільцевих напружень у піністому середовищі. Для розв'язання нестаціонарної задачі у випадку плоскої деформації для структурно неоднорідних матеріалів використана модель континууму Коссера. Ця модель дозволяє врахувати вплив зсувно-обертальної деформації мікрочастинок середовища. У рамках теорії пружності Коссера із застосуванням перетворень Фур'є для змінної часу та модифікації методу граничних інтегральних рівнянь розв'язання нестаціонарної задачі зводиться до системи сингулярних інтегральних рівнянь, де виділені компоненти, що визначають вплив деформації зсуву-обертання. Чисельні розрахунки виконано для пінополіуретану із закритими порами у випадку, коли середовище послаблено тунельною порожниною круглого перерізу під дією локалізованого імпульсного навантаження. Розроблений підхід може бути використаний для прогнозування механічної поведінки піноматеріалів під дією змінного у часі навантаження на основі аналізу розподілу динамічних напружень.

Ключові слова: *імпульсне навантаження, нестаціонарна плоска задача, теорія пружності Коссера.*

Introduction. In modern production, the creation and using of new structurally heterogeneous composite materials, which have low thermal conductivity, low density, long life and resistance to corrosive environments, is growing significantly. This applies foam materials based on polyurethane in particular.

Also, a lot of manufacturing processes associate with the necessity of applications of localized or concentrated mechanical effect on construction elements containing foam materials. Particularly urgent is the problem when transporting and moving construction elements which manufactured of foam materials. This necessitates the development of methods for calculating of the dynamic stress state of such construction elements, especially under the action of localized dynamic loads, which is due to technological and mechanical influences.

Literature review. In [1-3] shown that under the action of non-stationary distributed load investigation of stress state of foam materials (polyurethane, foam polimethacrylamide, etc.) can be performed in the framework of couple stress elasticity [4]. Under the action of this type of load, the vectors of macro-rotation of the medium and micro-rotation of particles can be considered almost identical.

Along with this, under the action of localized or concentrated forces, such an assumption gives significant errors in numerical calculations. Several numerical and experimental studies show that the distribution of stress fields under the action of concentrated load differs from the corresponding stress fields under distributed load [5-7]. To study such effects in foam materials, it is necessary accounting for the micro-rotations of the particles of the medium, which significantly differ in the vicinity of the application of the load and at remote points of the medium.

The studying of the dynamic stress state of structurally inhomogeneous media under the action of a localized load, the Cosserat moment elasticity is used [8, 9]. In the framework of Cosserat elasticity, the micro-rotation of each particle does not depend on its translational displacement. For describing the elastic behaviour of isotropic linear medium, six physical constants are used: two Lame's constants and four new constants characterizing the microstructure of the material. In the works of Erofeev V.I. [10] it is shown that for a quadratic-nonlinear medium the number of new constants increases to nine.

This necessitates the development of methods for calculating the dynamic stress state of such structurally inhomogeneous media, especially under the action of localized dynamic loads, which is due to technological and mechanical influences.

There are several works on the construction of solutions of these types of problems, which is associated with a significant complexity of both analytical and numerical calculations. Within the Cosserat continuum, Pal'mov V.A. [11] obtained a solution of the static problem of stress concentration near a circular hole. In the works of Sandru N., the problem of the action of a concentrated force and a concentrated moment in infinite elastic space is solved.

Therefore, the building of solutions of non-stationary problems under the action of concentrated forces, it is necessary to use methods accounting for the influence of defects on the stress state of the medium. Such methods include the method of boundary integral equations.

In [12] the development of the boundary integral equation method to solving time-domain problems of Cosserat elasticity was presented. The numerical implementation of the proposed approach is performed in [2, 12] for the case of a second exterior dynamic problem.

Therefore, **the aim of the research** is developing of boundary integral equation approach for the modelling of the dynamic stress state of foam media with tunnel defects under the action of localized impulse loads. For achieving this aim the apparatus of Cosserat elasticity is used.

Research methodology. Conctructive relations. For the solution of non-stationary problem for the case of structurally inhomogeneous medium we use apparatus of Cosserat elasticity where the motion equations of continuum are written as [1-3]:

$$\sigma_{ji,j} + X_i = \rho \ddot{u}_i, \tag{1}$$

$$\in_{kji} \sigma_{ij} + \mu_{jk,j} + Y_k = J \ddot{\phi}_k, \tag{2}$$

where σ_{ji} is the force stress, μ_{ji} is the couple stress, ρ is the material density, $X=\{X_i\}$ is the mass forces vector, $Y=\{Y_k\}$ is the couple forces vector, J is the inertia of unit volume rotation, $\in_{klm}$ is the permutation symbol, $u=\{u_i\}$ is the displacement vector, $\phi=\{\phi_k\}$ is the rotation vector. Functions u and ϕ are continuous functions. Here and further the Einstein summation convention is used. A comma at subscript denotes differentiation with respect to a coordinate indexed after the comma, i.e. $u_{j,i}=\partial u_j/\partial x_i$. Under the condition of plane strain indices vary from 1 to 2, and k=3.

The relations for determining force and couple stresses are written as [4]:

$$\begin{aligned} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij}, \\ \mu_{ji} &= (\gamma+\varepsilon)\kappa_{ji} + (\gamma-\varepsilon)\kappa_{ij} + \beta\kappa_{kk}\delta_{ij}, \end{aligned} \tag{3}$$

where α, β, γ, κ are the elastic constant required to describe an isotropic constrained Cosserat elastic solid, λ, μ are Lame parameters, $\gamma_{ij}=u_{i,j} - \in_{kji}\phi_k$ is the asymmetric deformation tensor, $\kappa_{ij}=\phi_{i,j}$ is the torsion bending tensor.

Statement of the problem. Let's consider a foam medium with tunnel cavity of sufficiently small diameter under a plane strain condition (Fig. 1, *a*). We denote a configuration of medium by Ω and the boundary of tunnel cavity with constant cross-section by L. The center of gravity is placed at the origin of Cartesian coordinate system $x_1Ox_2x_3$.

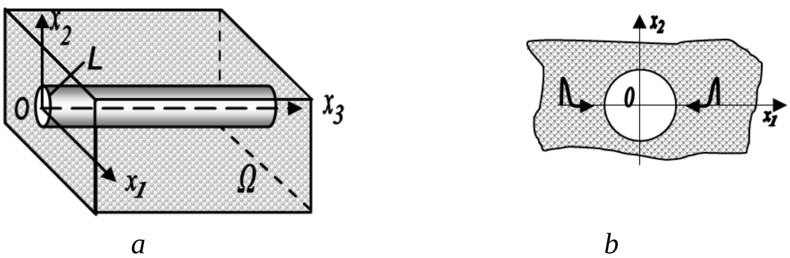

Fig. 1. **Model of the research object**

The problem consists in the determination of the dynamic stresses at foam medium with tunnel cavity of circular cross-section under the action of impulse localized loads, which are applied in internal points (Fig. 1, *b*).

The boundary conditions in Cosserat elasticity are written as:

$$\sigma_n\big|_L = \sigma(t,\delta), \quad \tau_{sn}\big|_L = \tau(t,\delta), \quad \mu_n\big|_L = 0, \tag{4}$$

where $\sigma(t, \delta)$ and $\tau(t, \delta)$ are the function that describes stresses, which are caused action of an impulse load $P(t)$, which is applied on the distance δ to the cavity's boundary in radial direction, n is the normal to the boundary of the cavity.

Solution of the non-stationary problem. For the solving non-stationary problem Fourier transforms for time variable [5] and modification boundary integral equation method [6] were used. The representations for transforms of displacements and microrotations are written [6]:

$$\hat{u}_i = \int_L p_j \cdot U_{ij}^* \, dL + \int_L m_k \cdot \Phi_{kj}^* \, dL + \int_\Omega X_j \cdot U_{ij}^* \, d\Omega + \int_\Omega Y_k \cdot \Phi_{kj}^* \, d\Omega, \tag{5}$$

$$\hat{\phi}_k = \int_L p_j \cdot U_{kj}^{**} \, dL + \int_L m_k \cdot \Phi_{kk}^{**} \, dL + \int_\Omega X_j \cdot U_{kj}^{**} \, d\Omega + \int_\Omega Y_k \cdot \Phi_{kk}^{**} \, d\Omega, \tag{6}$$

where $U_{ij}^*, U_{kj}^{**}, \Phi_{kj}^*, \Phi_{kk}^{**}$ are the fundamental functions for displacements and mictrorotations [6], p_j, m_k are unknown functions.

Inserting the representations of displacements (5) and microrotations (6) to the force and couple stress formulas (3) we obtain the integral dependencies for absent of couple forces:

$$\begin{aligned}
\hat{\sigma}_n &= \int_L \left(f_1\left(\mathbf{x}, \mathbf{x}^0\right) p_1 + f_2\left(\mathbf{x}, \mathbf{x}^0\right) p_2 + f_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 \right) dL(\mathbf{x}^0) + \int_\Omega \left(\Phi_1\left(\mathbf{x}, \delta\right) P + \Phi_2\left(\mathbf{x}, \delta\right) \bar{P} \right) d\Omega; \\
\hat{\tau}_n &= \int_L \left(g_1\left(\mathbf{x}, \mathbf{x}^0\right) p_1 + g_2\left(\mathbf{x}, \mathbf{x}^0\right) p_2 + g_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 \right) dL(\mathbf{x}^0) + \int_\Omega \left(\Gamma_1\left(\mathbf{x}, \delta\right) P + \Gamma_2\left(\mathbf{x}, \delta\right) \bar{P} \right) d\Omega; \\
\hat{\mu}_n &= \int_L \left(G_1\left(\mathbf{x}, \mathbf{x}^0\right) p_1 + G_2\left(\mathbf{x}, \mathbf{x}^0\right) p_2 + G_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 \right) dL(\mathbf{x}^0),
\end{aligned} \tag{7}$$

where $f_m, g_m, G_m, \Phi_k, \Gamma_k$ are known functions, which contain Bessel function of third kind.

Integration of functions f_m, g_m, G_m for the small value of argument leads to the singularity. To establish their characteristic we used the asymptotic expressions for Bessel function of the third kind for small values of the argument [7].

For the obtaining of integral equations of the problem the developed in [8] approach for the time-domain problem of classical theory of elasticity are used. For the determining of the unknown functions p_1, p_2, m_3 the Fourier transforms of boundary condition (4) are satisfied. We obtain the system of integral equations:

$$\begin{aligned}
&\frac{\mathrm{Re}(q)}{2} + \mathbf{v.p.}\int_L \left(f_1\left(\mathbf{x}, \mathbf{x}^0\right) q d\zeta + f_2\left(\mathbf{x}, \mathbf{x}^0\right) \bar{q} d\bar{\zeta} + f_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 dL \right) = -\int_\Omega \left(\Phi_1\left(\mathbf{x}, \delta\right) P + \Phi_2\left(\mathbf{x}, \delta\right) \bar{P} \right) d\Omega; \\
&\frac{\mathrm{Im}(q)}{2} \vartheta_1 + \mathbf{v.p.}\int_L \left(g_1\left(\mathbf{x}, \mathbf{x}^0\right) q d\zeta + g_2\left(\mathbf{x}, \mathbf{x}^0\right) \bar{q} d\bar{\zeta} + g_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 dL \right) = -\int_\Omega \left(\Gamma_1\left(\mathbf{x}, \delta\right) P + \Gamma_2\left(\mathbf{x}, \delta\right) \bar{P} \right) d\Omega; \\
&\frac{m_3}{2} I + \mathbf{v.p.}\int_L \left(G_1\left(\mathbf{x}, \mathbf{x}^0\right) q d\zeta + G_2\left(\mathbf{x}, \mathbf{x}^0\right) \bar{q} d\bar{\zeta} + G_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 dL \right) = 0,
\end{aligned} \tag{8}$$

where $pdL = -Iqd\zeta$, $p = p_1 + Ip_2$ is the unknown complex function, $\zeta = x_1^0 + Ix_2^0$, $\vartheta_1 = \left(1 - (\alpha / \mu)^2\right)$ is the constants. Here the integrals are understood in the sense of Cauchy principal value.

For the numerical solving of the system of integral equations (8) the algorithm [8] was applied. It was used method of mechanical quadrature and collocation method.

Calculations of the hoop stress transforms on the boundary of the cavity of the medium are performed by formulas [6]. Substituting in these formulas the potential representations for displacements (5) and microrotations (6), selecting irregular parts and completing limit transition the transforms of the hoop stresses on the boundary are written:

$$\hat{\sigma}_\theta = \frac{\mathrm{Re}(q)}{2} \vartheta_2 + \mathbf{v.p.}\int_L \left(h_1\left(\mathbf{x}, \mathbf{x}^0\right) q d\zeta + h_2\left(\mathbf{x}, \mathbf{x}^0\right) \bar{q} d\bar{\zeta} + h_3\left(\mathbf{x}, \mathbf{x}^0\right) m_3 dL \right); \tag{9}$$

where $h_k = h_k(\mathbf{x}, \mathbf{x}^0)$ are known functions, ϑ_2 is the constant ($\vartheta_2 = \nu / (1 - \nu)$ for plane strain).

Modified discrete Fourier transform is used for calculation of originals of the dynamic hoop and radial stresses [5].

Numeric calculation. Within the framework of the Cosserat elasticity, based on the developed boundary integral equation method [6] for elastic foam medium, we investigate the distribution of dynamic hoop stresses at the boundary of a circular cross-section cavity. Numerical calculations are performed for the case of the dynamic stress state of the medium is caused by the action of the system of concentrated non-stationary forces $P_j = P_j^{(1)} + iP_j^{(2)}$. Systems of these forces are applied at the internal points of the medium (a_{1j}, a_{2j}), $j = \overline{1, J}$. The vector of mass forces can be represented as:

$$X_1 = \sum_{j=1}^{N} P_{1j}\delta(x_1 - a_{1j})\delta(x_2 - a_{2j}), \quad X_2 = \sum_{j=1}^{N} P_{2j}\delta(x_1 - a_{1j})\delta(x_2 - a_{2j}). \tag{10}$$

The functions $P_{1j}(t)$ and $P_{2j}(t)$ in (10) describing change of impulse loads over the time are chosen in form [8]:

$$\varphi(\tilde{t}) = p_* \tilde{t}^{n_*} e^{-\alpha_* \tilde{t}}, \tilde{t} > 0, n_* \geq 0, \tag{11}$$

where $\tilde{t} = \frac{t \cdot c_l}{a}$ is a dimensionless time parameter, $c_l = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ is the speed of expansion wave, p^*, n^*, α^* are the constants, which are characterized duration of impulse load, a is some characteristic scale. For numerical calculations the value of characteristic scale is chosen as $a = a_1 \cdot 10^3$ m, where a_1 is radius of cavity cross-section.

For numerical calculations it was assumed that the concentrated dynamic forces were applied in the internal points of the foam medium on a distance of $(\pm 1.5a_1;\ 0)$ from the centre of gravity. Here a_1 is the radius of the cavity cross-section. The intensity change of the concentrated forces over the time was taken in the form (11) for the values of constants $\alpha_* = 4,\ n_* = 2,\ p_* = 30$. The calculation of normalized hoop stresses on the boundary of the cavity was performed for the interval of a dimensionless time parameter $\tilde{t} \in [0;\ 20]$ with impulse duration $t_*=2$.

Numerical calculations were performed for the case of the cavity cross-section has diameter is *1 cm*, which corresponds to the conditions of plane strain. For ensuring the accuracy of the calculations, 120 points on the boundary of the cavity cross- section were selected [8].

Dynamic hoop stresses were calculated for polymethacrylamide closed-cell foam (Rohacell WF300) [1]. For the numerical determining of the dynamic hoop stresses the physic characteristics of the closed-cell foam were chosen [1]: cell size $d = 0.6\ mm$, material density $\rho = 380\ kg / m^3$, Poisson's ratio $\nu = 0.13$, Young's modulus $E = 637\ MPa$, characteristics of material microstructures are $\alpha = 2.85$ MPa, $\gamma = 182$ N, $\varepsilon = 494$ N.

Fig. 2, *a* shows the results of numerical calculations of the normalized dynamic hoop stresses at the boundary of the circular cross-section tunnel cavity for the different values of the angle θ. Here, curve 1 corresponds to the stress values at point $\theta = 90°$ on the boundary of the cross-section of the cavity, curve 2 and 3 correspond to the case $\theta = 45°$ and $\theta = 0$ accordingly. It was assumed that $\bar{\sigma}_\theta = \sigma_\theta / \sigma_0$, where σ_0 is the maximum values of stresses arising from the action of impulse concentrated forces in the medium in the absence of a hole.

For the evaluation of the efficiency of using the model of Cosserat elasticity for the investigation of the dynamic stress state of the foam materials, we compare the results obtained based on moment theory elasticity equations with the results, which are calculated based on classical elasticity of theory. We use the approach described in [9] for the calculation of dynamic stress distribution in the framework of classical elasticity. For the numerical calculations, we used the same values of the physic characteristics of polymethacrylamide foam without accounting for the characteristics of the microstructure of the material.

According calculation results for clased-cell Rohacell WF300 foam [1] in the framework of classical theory of elasticity are shown in Fig. 2, *b*. Here, curves 1-3 correspond to the same values as those chosen for the Cosserat elasticity.

The comparing of the curves of Fig. 2 is shown that distribution of normalized hoop stresses on the boundary of the tunnel cavity differs not only by them magnitudes but also the character of their distributions. However, maximum values of the hoop dynamic stresses arise around same points on the boundary of the cavity.

Based on the numerical calculation results it was establish that in the case of accounting for the influence of the microstructure of the material in the framework of Cosserat elasticity, the maximum values of the normalized hoop dynamic stresses on the boundary of the tunnel cavity are smaller in 1.5 times than without accounting for the influence of the microstructure of medium. Such vibration-absorbing properties of foam materials are confirmed on the basis of experimental studies [18].

Fig. 2, *a* shows, that the attenuation of wave processes occurring in a foam medium under the action of a non-stationary loads can be more correctly described on the basis of the equations of the theory of moment elasticity (Cosserat continuum). Such effects cannot be described within the framework of classical elasticity of theory.

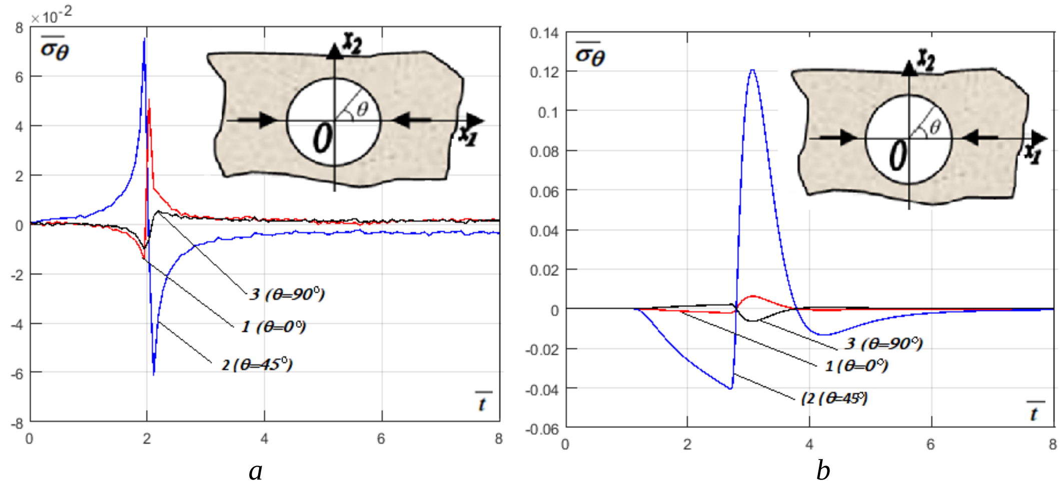

Fig. 2. **The distribution of normalized hoop stresses on the boundary of a circular tunnel cavity in foam polyurethane WF 300 (Cosserat elasticity –*a*, classical elasticity - *b*)**

Conclusions. Basic on the developing for Cosserat elasticity modify boundary integral equation method [6] and according to developed approach in the framework of classical elasticity [10] the comparison of the distribution of normalized dynamic hoop stresses in foam materials is performed. The results of the numerical calculations confirm the necessity of using the refined theories of the continuum mechanics (moment theory of elasticity – Cosserat elasticity) for investigation the dynamic stress state of foam materials under the action of time-variable concentrated loads.

Based on the numerical calculations, it is established that the distributions of the normalized hoop stress in foam media with accounting for the influence of material microstructure (Cosserat elasticity) and without accounting for this influence (classical elasticity) have not only distinctive types of stress distribution but also by thier magnitudes. Besides, using the apparatus of Cosserat elasticity permit more correctly to describe vibration-absorbing properties of foam materials, which is confirmed by several experimental studies [1-3, 10].

An adaptive approach can be used for the investigation of the effect of time-variable load on the distribution of hoop stresses in foam media and for prediction of the mechanical behaviour of these types of materials.

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