УДК 621.01:621-868 DOI 10.36910/775.24153966.2025.81.6

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АНАЛІЗ ДИНАМІЧНИХ ХАРАКТЕРИСТИК РОБОЧОГО ОРГАНА ВІБРАЦІЙНОГО ТРУБЧАСТОГО КОНВЕЄРА З УРАХУВАННЯМ ПОПЕРЕЧНИХ КОЛИВАНЬ

У статті представлено аналіз динамічних характеристик кінцевого ефектора вібраційного трубчастого конвеєра з урахуванням поперечних коливань. Методика полягає в отриманні та розв'язанні загального диференціального рівняння поперечних коливань з використанням функцій Крилова. Такий підхід дає змогу точно визначити режими та амплітуди коливань. Новизна дослідження полягає у врахуванні впливу пружної основи на поведінку кінцевого ефекту, що дозволяє визначити критичні частоти паразитних коливань. Отримані результати мають практичну цінність, оскільки підвищують стабільність транспортування матеріалу та надійність вібраційних трубчастих конвеєрів у промисловому застосуванні.

Ключові слова: *вібраційний трубчастий конвеєр, поперечні коливання, вібраційний режим, паразитні коливання, пружна основа, функції Крилова, динамічний аналіз, частота коливань.*

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ANALYSIS OF DYNAMIC CHARACTERISTICS OF THE WORKING BODY OF A VIBRATORY TUBULAR CONVEYOR CONSIDERING TRANSVERSE VIBRATIONS

The article presents an analysis of the dynamic characteristics of the vibration tubular conveyor's end-effector with consideration of transversal vibrations. The methodology involves deriving and solving the general differential equation of transversal vibrations using Krylov's functions. This approach enables accurate determination of vibration modes and amplitudes. The novelty of the research lies in accounting for the influence of the elastic foundation on the behaviour of the end-effector, which allows identifying critical parasitic vibration frequencies. The obtained results provide practical value by enhancing the stability of material transportation and improving the reliability of vibration tubular conveyors in industrial applications.

Keywords: *Vibratory tubular conveyor, transverse vibrations, vibration mode, parasitic vibrations, elastic foundation, Krylov's functions, dynamic analysis, vibration frequency.*

Introduction. A critical challenge in designing and practically implementing vibratory conveying systems, as highlighted in recent studies [6, 7, 11] is ensuring uniform material transportation along the entire length of the end-effector (EE). Achieving uniform transportation speed is essential for improving the efficiency and reliability of industrial processes involving bulk materials. This challenge becomes particularly significant when dealing with elongated end-effectors subjected to transverse excitation forces, common in tubular conveyor designs. Such transverse excitation forces induce not only the desired beneficial vibrations required for material transportation but also additional unwanted parasitic transverse vibrations. The superposition of intended and parasitic vibrations can lead to uneven material flow along the conveyor's EE, and in extreme cases, may cause localized disruptions or even complete stoppage of transportation. Despite considerable research in vibratory conveyor systems, the interaction between transverse vibrations and the elastic foundation, particularly for elongated conveying elements, remains insufficiently explored, necessitating further analytical and experimental investigations.

Problem statement. During the design of vibratory conveying systems, accurate calculation of transverse vibration amplitudes along the entire length of the end-effector (EE) is essential to ensure stable and uniform material transportation. Uneven vibration amplitudes can cause zones of inefficient or halted transport, directly affecting conveyor performance. This issue is particularly relevant for vibratory tubular conveyors (VTC) [1], characterized by elongated structures on elastic foundations. Generalized methods available in the literature often fail to fully capture specific practical features, such as stiffness distribution variations and local parasitic vibration phenomena. Thus, developing refined analytical methods to accurately predict vibration modes and identify critical frequencies remains an important practical task for improving conveyor reliability. A typical example illustrating this challenge is presented in Fig. 1.

Literature review. The literature [3] provides well-developed methodologies for calculating transverse vibrations of elongated beams with distributed masses. However, the applied aspects of calculating elongated end-effectors on an elastic foundation, which are characteristic of vibratory conveyor structures, remain insufficiently explored.

Objectives of research. This study aims to perform an analytical calculation of the transverse vibrations of the elongated end-effector of a vibratory tubular conveyor with an electromagnetic drive mounted on an elastic foundation.

The primary focus is placed on investigating the influence of the elastic mounting characteristics on the vibration modes and amplitudes of the end-effector.

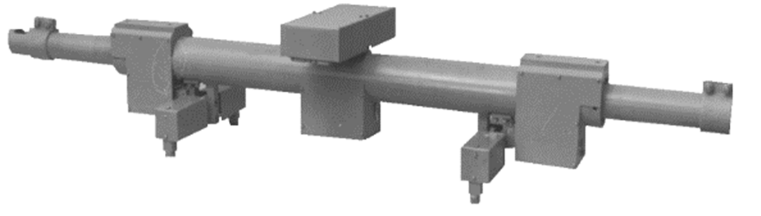

Fig. 1. **Vibratory tubular conveyor**

Thus, determining the vibration amplitudes and modes of the end-effector supported by elastic elements is the main objective of this article.

Main material presentation. The structural diagram of the vibratory tubular conveyor (VTC) is presented in Fig. 2.

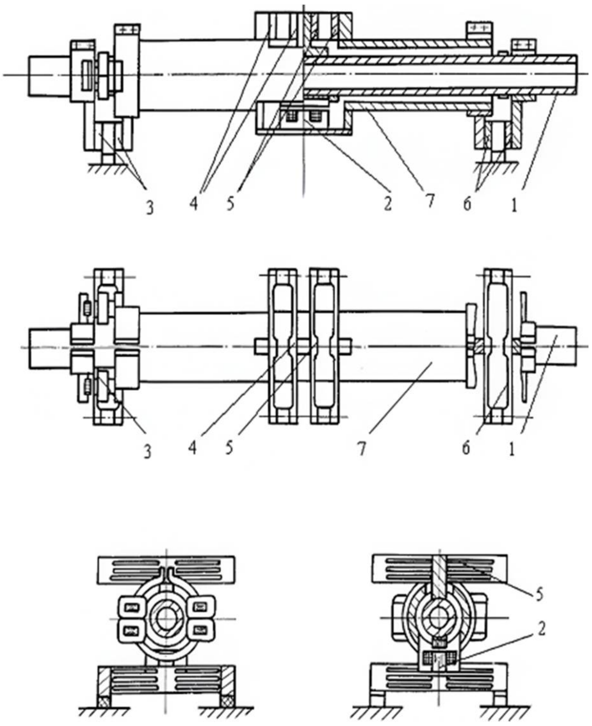

Fig. 2. **Structural diagram of the vibratory tubular conveyor**

The length of the VTC's end-effector is $l = 2\ m$, with an outer cross-sectional diameter of $d_{out} = 74.5$ mm and an inner diameter of $d_{in} = 68$ mm. The VTC is designed with independent excitation of the end-effector's vibrations in both vertical and horizontal directions. The transverse excitation force $P(t)$ generated by the electromagnetic exciter (2), is applied at the midpoint of the end-effector. Along with the intended beneficial transverse vibrations that ensure uniform material transportation, undesirable parasitic vibrations may occur, disrupting the stability of material flow. The end-effector is mounted using four spring blocks (3, 4, 5, 6) attached to the reactive element (7).

The computational model of the VTC end-effector with elastic suspensions, supports, and an applied force is shown in Fig. 3. The elongated end-effector is supported by three elastic elements with stiffness values c_1, c_2, c_3. The stiffness c_2 is considered as the total stiffness of spring blocks 4 and 5 (Fig. 2) since the distance between these blocks is minimal.

The motion model of the end-effector, treated as an elastic beam with free ends and distributed mass, is well-known from the literature, for example, in [4]. Given the significant length of the VTC's end-effector compared to its transverse dimensions, the general equation of free vibrations for the end-effector as an elastic beam with distributed mass, neglecting the rotational inertia of the cross-section, can be written as follows:

$$\frac{\partial^2 y}{\partial t^2} + c^2 \frac{\partial^4 y}{\partial x^4} = 0, \tag{1}$$

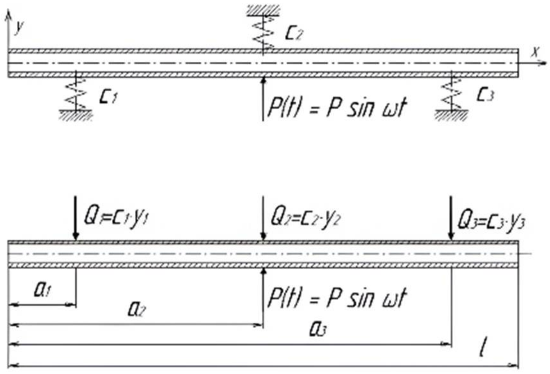

Fig. 3. **Computational model of the vibratory tubular conveyor end-effector on elastic suspension under load conditions**

where y is the displacement coordinate of the VTC end-effector points in the transverse (vertical) direction; x is the coordinate of the VTC end-effector points in the horizontal direction; t is time

$$c = \sqrt{\frac{EJ}{\mu}}$$

Here, EJ is the flexural rigidity (E is the modulus of elasticity, and J is the moment of inertia of the VTC end-effector's cross-section relative to the neutral axis perpendicular to the vibration plane); μ is the mass per unit length of the end-effector.

The solution to this equation for principal vibrations takes the form:

$$y(x,t) = \varphi(x)\sin(pt + \alpha), \tag{2}$$

where $\varphi(x)$ is the vibration mode function, p is the natural frequency of vibrations, and α is the phase shift angle of the vibration mode.

The integral function $\varphi(x)$ is equal to:

$$\varphi(x) = A\cos kx + B\sin kx + Cch\ kx + Dsh\ kx, \tag{3}$$

where A, B, C, D are arbitrary constants determined from boundary conditions; $k^4 = \frac{\mu p^2}{EJ}$.

Due to the complexity of direct integration for boundary-value problems in beam vibrations, Krylov's functions method, known for simplifying integral forms and facilitating analytical and numerical solutions, is applied [4]. Thus, we rewrite the integral using Krylov's functions as follows:

$$\varphi(x) = AS(kx) + BT(kx) + CU(kx) + DV(kx), \tag{4}$$

where Krylov's functions $S(kx), T(kx), U(kx), V(kx)$ are defined as follows:

$$S(kx) = \frac{1}{2}(chkx + coskx),\ T(kx) = \frac{1}{2}(shkx + sinkx),$$
$$U(kx) = \frac{1}{2}(chkx - coskx),\ V(kx) = \frac{1}{2}(shkx - sinkx) \tag{5}$$

In this case, since the VTC end-effector has no fixed supports at its ends, the boundary conditions are as follows:

$$\varphi''(0) = \varphi'''(0) = 0, \tag{6a}$$
$$\varphi''(0) = \varphi'''(l) = 0. \tag{6b}$$

The integral that satisfies the condition at $x = 0$ (Eq. 6a) is:

$$\varphi(x) = AS(kx) + BT(kx). \tag{7}$$

For $x = l$:

$$\varphi(l) = AS(kl) + BT(kl) + \Phi_1 + \Phi_2 + \Phi_3 + \Phi_P, \tag{8}$$

where $\Phi_1 + \Phi_2 + \Phi_3 + \Phi_P$ are the values of Krylov's partial integrals, which for our conditions are as follows

$$\begin{cases} \Phi_1(x) = \dfrac{c_1 y_1}{k^3 EJ} V\left[k\left(l - a_1\right)\right], \\ \Phi_2(x) = \dfrac{c_2 y_2}{k^3 EJ} V\left[k\left(l - a_2\right)\right], \\ \Phi_3(x) = \dfrac{c_3 y_3}{k^3 EJ} V\left[k\left(l - a_3\right)\right]. \end{cases} \quad (9)$$

where Φ_P is the disturbing harmonic force, $\Phi_P = -\dfrac{P}{k^3 EJ} V\left[k\left(l - a_2\right)\right]$.

By satisfying Eq. (6b), Eq. (8) can be rewritten as follows:

$$AU(kl) + BV(kl) + \frac{c_1 y_1}{k^3 EJ} T\left[k\left(l - a_1\right)\right] + \frac{c_2 y_2}{k^3 EJ} T\left[k\left(l - a_2\right)\right] + \\ + \frac{c_3 y_3}{k^3 EJ} T\left[k\left(l - a_3\right)\right] - \frac{P}{k^3 EJ} T\left[k\left(l - a_2\right)\right] = 0; \quad (10)$$

$$AT(kl) + BU(kl) + \frac{c_1 y_1}{k^3 EJ} S\left[k\left(l - a_1\right)\right] + \frac{c_2 y_2}{k^3 EJ} S\left[k\left(l - a_2\right)\right] + \\ + \frac{c_3 y_3}{k^3 EJ} S\left[k\left(l - a_3\right)\right] - \frac{P}{k^3 EJ} S\left[k\left(l - a_2\right)\right] = 0.$$

Based on Eqs. (7) and (8), a system of three equations is obtained, which considers the displacements y_1, y_2, y_3:

For $x = a_1$: $AS\left(ka_1\right) + BT\left(ka_1\right) - y_1 = 0$

For $x = a_2$: $AS\left(ka_2\right) + BT\left(ka_2\right) + \dfrac{c_1 y_1}{k^3 EJ} V\left[k\left(a_2 - a_1\right)\right] - y_2 = 0;$ (11)

For $x = a_3$:

$$AS\left(ka_3\right) + BT\left(ka_3\right) + \frac{c_1 y_1}{k^3 EJ} V\left[k\left(a_3 - a_1\right)\right] + \frac{c_2 y_2}{k^3 EJ} V\left[k\left(a_3 - a_2\right)\right] - \frac{P}{k^3 EJ} V\left[k\left(a_3 - a_2\right)\right] - y_3 = 0$$

Thus, the system of equations describing the natural vibration mode of the VTC end-effector, considering Eqs. (10) and (11), has the following form:

$$\begin{cases} AS\left(ka_1\right) + BT\left(ka_1\right) - y_1 = 0; \\ AS\left(ka_2\right) + BT\left(ka_2\right) + \dfrac{c_1 y_1}{k^3 EJ} V\left[k\left(a_2 - a_1\right)\right] - y_2 = 0; \\ AS\left(ka_3\right) + BT\left(ka_3\right) + \dfrac{c_1 y_1}{k^3 EJ} V\left[k\left(a_3 - a_1\right)\right] + \\ + \dfrac{c_2 y_2}{k^3 EJ} V\left[k\left(a_3 - a_2\right)\right] - \dfrac{P}{k^3 EJ} V\left[k\left(a_3 - a_2\right)\right] - y_3 = 0; \\ AU(kl) + BV(kl) + \dfrac{c_1 y_1}{k^3 EJ} T\left[k\left(l - a_1\right)\right] + \dfrac{c_2 y_2}{k^3 EJ} T\left[k\left(l - a_2\right)\right] + \\ + \dfrac{c_3 y_3}{k^3 EJ} T\left[k\left(l - a_3\right)\right] - \dfrac{P}{k^3 EJ} T\left[k\left(l - a_2\right)\right] = 0; \\ AT(kl) + BU(kl) + \dfrac{c_1 y_1}{k^3 EJ} S\left[k\left(l - a_1\right)\right] + \dfrac{c_2 y_2}{k^3 EJ} S\left[k\left(l - a_2\right)\right] + \\ + \dfrac{c_3 y_3}{k^3 EJ} S\left[k\left(l - a_3\right)\right] - \dfrac{P}{k^3 EJ} S\left[k\left(l - a_2\right)\right] = 0. \end{cases} \quad (12)$$

The analytical solution of Eq. (12) by means of the constants A and B is complex and impractical due to the high-order equations and nonlinear boundary conditions. Therefore, numerical methods implemented in Mathcad software were utilized, providing efficient and accurate solutions that sufficiently meet practical engineering requirements.

The unknown constants can be determined from the following equation:

$$\boldsymbol{X} = \boldsymbol{C}^{-1}\boldsymbol{P}, \tag{13}$$

where X is the column vector of unknowns, C is the coefficient matrix, and P is the column vector of external forces.

In expanded form, Eq. (13) can be rewritten as follows:

$$\begin{bmatrix} A \\ B \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} S(ka_1) & T(ka_1) & -1 & 0 & 0 \\ S(ka_2) & T(ka_2) & \frac{c_1}{k^3EJ}V\left[k(a_2-a_1)\right] & -1 & 0 \\ S(ka_3) & T(ka_3) & \frac{c_1}{k^3EJ}V\left[k(a_3-a_1)\right] & \frac{c_2}{k^3EJ}V\left[k(a_3-a_2)\right] & -1 \\ U(kl) & V(kl) & \frac{c_1}{k^3EJ}T\left[k(l-a_1)\right] & \frac{c_2}{k^3EJ}T\left[k(l-a_2)\right] & \frac{c_3}{k^3EJ}T\left[k(l-a_3)\right] \\ T(kl) & U(kl) & \frac{c_1}{k^3EJ}S\left[k(l-a_1)\right] & \frac{c_2}{k^3EJ}S\left[k(l-a_2)\right] & \frac{c_3}{k^3EJ}S\left[k(l-a_3)\right] \end{bmatrix}^{-1} \times$$

$$\times \begin{bmatrix} 0 \\ 0 \\ \frac{P}{k^3EJ}V\left[k(a_3-a_2)\right] \\ \frac{P}{k^3EJ}T\left[k(l-a_2)\right] \\ \frac{P}{k^3EJ}S\left[k(l-a_2)\right] \end{bmatrix}. \tag{13}$$

The vibration mode function $\varphi(x)$ in separate sections can be generally written as follows:

For $0 \le x \le a_1$: $\varphi(x) = AS(kx) + BT(kx)$.

For $a_1 \le x \le a_2$: $\varphi(x) = AS(kx) + BT(kx) + \frac{c_1 y_1}{k^3EJ}V\left[k(l-a_1)\right]$;

For $a_2 \le x \le a_3$:

$$\varphi(x) = AS(kx) + BT(kx) + \frac{c_1 y_1}{k^3EJ}V\left[k(l-a_1)\right] + \frac{c_2 y_2}{k^3EJ}V\left[k(l-a_2)\right] - \frac{P}{k^3EJ}V\left[k(l-a_2)\right] \tag{14}$$

For $a_3 \le x \le l$:

$$\varphi(x) = AS(kx) + BT(kx) + \frac{c_1 y_1}{k^3EJ}V\left[k(l-a_1)\right] + \frac{c_2 y_2}{k^3EJ} \times$$

$$\times V\left[k(l-a_2)\right] + \frac{c_3 y_3}{k^3EJ}V\left[k(l-a_3)\right] - \frac{P}{k^3EJ}V\left[k(l-a_2)\right].$$

The final equation of system (14) is the general equation for the vibration mode of the VTC end-effector over its entire length. Thus, by determining the numerical values of the unknowns A, B, y_1, y_2, y_3 from Eq. (13) for given parameters of a mechanical vibration system with distributed mass and substituting them into the vibration mode equation for the end-effector, the mode shape of the end-effector at the specified excitation frequency ω can be obtained.

For the following parameters:

$$l = 2m;\ a_1 = 0.55m; a_2 = 1m; a_3 = 1.45m;\ E = 2 \cdot 10^{11}\ Pa;\ c_1 = 268000\ N/m;$$

$$\mu = 7kg/m;\ J = 5.3 \cdot 10^{-7} \mathrm{m}^{-4};\ P = 200N;\ \omega = 157\ ^{rad}/_{s};\ c_2 = 482142\ N/m;$$

$$c_3 = 268000\ N/m.$$

the vibration mode function $\varphi(x)$ takes the form shown in Fig. 4a.

By setting the excitation frequency to $\omega = 3000\ ^{rad}/_{s}$, the vibration mode of the end-effector is presented as shown in Fig. 4b.

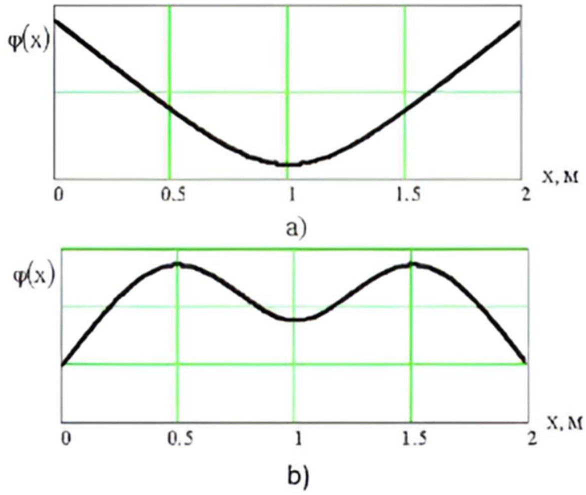

Fig. 4. **Vibration modes of the VTC end-effector for the excitation frequency (a)** $\omega = 157\ {}^{rad}/_{s}$**; (b)** $\omega = 3000\ {}^{rad}/_{s}$

Conclusions. Based on the conducted research, the above-described mathematical model of the end-effector allows for the analysis of vibration modes as well as their amplitudes at each point of the end-effector. The developed mathematical model is an effective tool for determining parasitic dangerous vibration frequencies of the end-effector, which contributes to improving the operational reliability of the structure. The obtained results enable enhancing the accuracy of designing vibratory conveyors with elongated end-effectors on an elastic foundation, which will improve their operational reliability. These studies, as well as the identification of the most optimal attachment options for elastic elements, are planned to be further investigated in future scientific work.

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