УДК: 539.3 DOI 10.36910/775.24153966.2024.77.6

Ю.Ю. Жигуц[1], В.Д. Рудь[2], В.Ф. Лазар[3]

ДВНЗ «Ужгородський національний університет»[1]
Луцький національний технічний університет[2]
Мукачівський державний університет[3]

РОЗВ'ЯЗОК КОНТАКТНОЇ ЗАДАЧІ ПРО ПЕРЕДАЧУ ЗОСЕРЕДЖЕНОГО НАВАНТАЖЕННЯ

В роботі розглянуто розв'язок плоскої контактної задачі на основі лінеаризованої теорії пружності про передачу навантаження за допомогою нескінченного неоднорідного стрингера до двох защемлених одною гранню пружних смуг при наявності початкових (або залишкових) напружень. Авторами виконано дослідження у загальному вигляді для дуже великих початкових деформацій і розглянуто деякі варіанти теорії малих початкових деформацій при умові довільної структури пружного потенціалу. З використанням інтегрального перетворення Фур'є вдалося одержати розв'язок основних інтегро-диференційні рівнянь і представити його у вигляді квазірегулярних нескінченних систем. Крім цього досліджено вплив існуючих початкових (залишкових) напружень для пружних смуг на закон розподілу дійсних контактних напружень за лінією з нескінченним неоднорідним стрингером.

Наявні початкові напруження у таких системах смуг призводять до якісної зміни закону розподілу контактних напружень, а саме при стискуванні контактні напруження суттєво зменшуються (а для розтягування – збільшуються), таким чином переміщення при стискування суттєво зростають, а при розтягуванні – зменшуються. Кількісний характер впливу початкових напружень у високоеластичних матеріалах при порівнянні з жорсткішими матеріалами має аналогічний характер, що і якісний.

Ключові слова: лінеаризована теорія пружності, початкові (залишкові) напруження, контактні задачі, інтегральне перетворення Фур'є.

Yu. Zhiguts, V. Rudj, V. Lazar

SOLUTION OF THE CONTACT PROBLEM ON THE TRANSMISSION OF A CONCENTRATED LOAD

The paper considers the solution of the plane contact problem based on the linearized theory of elasticity about the load transfer using an infinite non-uniform stringer to two elastic strips clamped by one face in the presence of initial (or residual) stresses. The authors performed a general study for very large initial deformations and considered some variants of the theory of small initial deformations under the condition of an arbitrary structure of the elastic potential. Using the integral Fourier transform, it was possible to obtain the solution of the main integro-differential equations and present it in the form of quasi-regular infinite systems. In addition, the influence of existing initial (residual) stresses for elastic strips on the distribution law of actual contact stresses along a line with an infinite non-homogeneous stringer was investigated.

The initial stresses present in such strip systems lead to a qualitative change in the law of distribution of contact stresses, namely, during compression, the contact stresses significantly decrease (and for stretching, they increase), thus, displacements during compression significantly increase, and during stretching, they decrease. The quantitative nature of the influence of the initial stresses in highly elastic materials when compared with stiffer materials has a similar nature to the qualitative one.

Key words: contact problems, the linearized elasticity theory resiliency, resilient protective strap, initial (residual) tension, initial deformations.

Introduction. Despite the large amount of literature in which contact problems of the linearized theory of elasticity are considered [1,2], research on the problems of elastic contact interaction of bodies with initial stresses in our country and abroad appeared relatively recently. Only recently, the study of the contact interaction of prestressed bodies has gained special interest in connection with the introduction into practice of new artificial materials that can withstand large initial deformations. In practice, residual stresses (for example, technological ones) are artificially created to compensate or adjust them in structural elements during operation, as well as to increase strength and stability. This is justified by the fact that when solving the problem of contact interaction of elastic bodies with initial stresses, the linear theory of elasticity used cannot take into account the influence of these stresses. To ensure the calculation of the entire process, they can be taken into account using the linearized theory of elasticity [1,3-6]. Taking into account the initial stresses within the framework of the linearized theory of elasticity leads to new formulations of problems of the interaction of deformed bodies, which differ significantly from the formulations of classical problems of the theory of elasticity. And although when solving these problems, the system of basic differential equations, expressions that are used to determine the components of the stress tensor and for which the entire structure of the boundary conditions differs significantly from the corresponding systems of the described equations of the stress tensor of the theory of elasticity, by their nature and structure, they are related to ordinary contact mixed problems [6-8]. This paper shows a possible variant of setting up and using a general method of solving the problem for an arbitrary form of elastic potentials, which is provided in a general form for theories of large initial strains, as well as the use of some types of problems from the theory of small initial strains.

When setting the specified tasks in the cited works, an assumption was made, namely, the contact interaction of bodies with initial stresses and elastic stringers is carried out immediately after the initial stress state occurs; external loads in the elastic overlay cause a perturbation of the stressed deformation state with a value much smaller than the value of the initial stressed state; the initial stress state of one of the interacting bodies has a structure that can be considered homogeneous; the solution of linearized problems of the theory of elasticity related to the contact interaction of stressed bodies and overlays is unique [6].

In fact, in the work using the relations of the linearized theory of elasticity [1, 8-10] solutions to the problem of the contact interaction of a heterogeneous stringer with pre-stressed tapes are proposed. The study was conducted in a general form for compressible and incompressible bodies using the theory of initial deformations with an arbitrary structure of the elastic potential.

Using [1,4,6,10], we will perform the calculation in the coordinates of the initial deformed state, y_i, which are related to the Lagrangian coordinates x_i by the relations $y_i = \lambda_i x_i$, $(i = 1{,}2)$ where λ_i – are the elongation coefficients that determine the change in the coordinates of the initial state in the directions of the coordinate axes. When conditions 1–4 are fulfilled in the area of contact $L_k\{a_k, b_k\}$ for elastic overlays and an elastic strip with residual stresses, the boundary conditions apply at

$$y_2 = 0\ u(y_1) = u_1(y_1);\, v(y_1) = u_2(y_1);\, \forall(y_1) \in L_k. \quad (1)$$

$$\frac{\mathrm{du}}{\mathrm{dy_1}} = \frac{\mathrm{du_1}}{\mathrm{dy_1}};\, \frac{\mathrm{dv}}{\mathrm{dy_1}} = \frac{\mathrm{du_2}}{\mathrm{dy_1}};\, \forall(\mathrm{y_1}) \in \mathrm{L_k} \quad (2)$$

Boundary conditions (1) and (2) together with conditions (1–4)

$$p = \int_{a_k}^{y_1} \tau(t)dt, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

complete the formulation of linearized problems about the contact interaction of elastic overlays ($a_k = -\infty; b_k = +\infty$) that reinforce the elastic strip.

Statement of the problem and basic solving equations. Let the infinite elastic strips of thickness with the initial stresses are pinched by faces , and their other faces are connected to each other by an infinite inhomogeneous elastic stringer of small thickness h (Fig. 1).

The infinite prestressed strips reinforced H in this way are under the action of the distributed horizontal forces $y_2 = \pm H$ of intensity applied to the connecting infinite non-homogeneous stringer according to h (Fig. 1). It is necessary to establish the laws of distribution of normal $p(y_1)$ and horizontal $q(y_1)$ stresses in the area of contact.

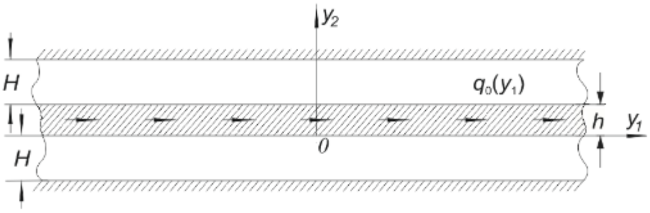


Fig. 1. **Diagram of the distribution of horizontal forces applied to a prestressed strip reinforced with a stringer**

When the area of contact with the stringer is investigated, we assume that under the action applied load and tangential stresses, it stretches or compresses like a rod in a uniaxial stress state [11]. We also assume that along the horizontal axis there are vertical elastic movements of steel. The last assumption is due to the small thickness of the stringer, since its changes from point to point in the process of deformation are insignificant and can be neglected.

Let us denote the intensities of normal and tangential contact stresses acting along the line of connection of the stringer with elastic prestressed strips $p(y_1)$ and $q(y_1)$, and $u_1(y_1)$ vertical and $u_2(y_1)$ horizontal displacements respectively.

Let's move on to obtaining the main systems of solving equations for the given problem. To this end, we first consider the equilibrium of the stringer.

From the condition of equilibrium $(-\infty, x)$, we get parts of the stringer

$$\sigma_{y_1 y_1}{}^{(y_1)} = \frac{1}{h}\int_{-\infty}^{y_1} [q(t) - q_0\,(t)]dt, (-\infty < y_1 < \infty). \quad (4)$$

This means that the cross-section of the stringer is rectangular, the width of which is equal to unity, and the $\sigma_{y_1 y_1}$ axial stress in the direction of the axis. According to Hooke's law, we will determine the axial tension and equilibrium of the stringer Oy_1.

From the condition of equilibrium, we get parts of the stringer

$$\sigma_{y_1 y_1(y_1)} = E_1 \varepsilon_{y_1, y_1}(y_1), \quad (5)$$

$$\text{where } \varepsilon_{y_1 y_1(y_1)} = \frac{du(y_1)}{dy_1}, \quad (6)$$

here $u(y_1)$ - horizontal displacements of points of the elastic stringer, E_1 - modulus of elasticity of the stringer.

Taking into account (4) – (6), we find

$$\frac{du(y_1)}{dy_1} = \frac{1}{E_1 h} \int_{-\infty}^{y_1} [q(t) - q_0(t)] dt, (-\infty < y_1 < \infty). \quad (7)$$

With the established assumption that the stringer bends in the vertical direction, it is possible to write

$$D \frac{d^4 v(y_1)}{dy_1^4} = p(y_1) - p_0(y_1), (-\infty < y_1 < \infty). \quad (8)$$

here $v(y_1)$ – vertical movements of stringer points; D – bending stiffness of the stringer; $p_0(y_1), p(y_1)$ – intensity of vertical forces.

Existing conditions at the line of contact of stringer with elastic bands

$$u(y_1) = u_1(y_1), v(y_1) = u_2(y_1), \forall y_1 \in (-\infty < y_1 < \infty), \quad (9)$$

where $u_1(y_1), u_2(y_1)$ is the displacement of points in elastic bands with initial stresses. Let's determine the law of distribution of normal and tangential contact stresses along the connection line of the stringer with the prestressed tapes.

To determine the movements and stresses along the line of contact of the stringer with the strips, we will set the boundary conditions of the problem for the edges of the strips. free from pinching with initial stresses from the force P applied at an angle α_0 [4,6,11]

$$\tilde{Q}_{22}(y_1, 0) = -P\delta(y_1) \sin \alpha_0 ; \tilde{Q}_{11}(y_1, 0) = -P\delta(y_1) \cos \alpha_0; \quad (10)$$

$$u_1(y_1 - t) = 0; u_2(y_1 - t) = 0; (-\infty < y_1 < \infty), \quad (11)$$

where is the Dirac delta function.

As a result of solving the given problem, the impact function from the action of the tangential force (at $\alpha_0 = 0$) for equal roots of the characteristic equation [1] ($n_1 = n_2$) are as follows:

$$h_{21}(y_1) = \frac{1}{\pi} \int_0^\infty H_{21}(\alpha) \sin \alpha\, y_1 d\alpha , \quad (12)$$

$$h_{22}(y_1) = \frac{1}{\pi} \int_0^\infty H_{22}(\alpha) \cos \alpha\, y_1 d\alpha.$$

For unequal roots ($n_1 \neq n_2$) we can write

$$h_{21}(y_1) = \frac{1}{\pi} \int_0^\infty \widetilde{H}_{21}(\alpha) \sin \alpha\, y_1 d\alpha, \quad (13)$$

$$h_{22}(y_1) = \frac{1}{\pi} \int_0^\infty \widetilde{H}_{22}(\alpha) \cos \alpha\, y_1 d\alpha.$$

For unequal roots ($n_1 \neq n_2$) we can write

$$h_{21}(y_1) = \frac{1}{\pi} \int_0^\infty \widetilde{H}_{21}(\alpha) \sin \alpha\, y_1 d\alpha, \quad (14)$$

$$h_{22}(y_1) = \frac{1}{\pi} \int_0^\infty \widetilde{H}_{22}(\alpha) \cos \alpha\, y_1 d\alpha.$$

The kernels $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)$ respectively have the form [1,12,13]:

for $n_1 = n_2$

$$H_{21}(\alpha) = m_0[-(s+1)(s_1 \xi(\alpha) - \alpha\phi_1) + ch^2 \alpha\phi_1 - s_1 sh^2 \alpha\phi_1 - s] =$$

$$= m_0[-(s+1)(s_1 sh\alpha\phi_1 ch\alpha\phi_1 - \alpha\phi_1) + ch^2 \alpha\phi_1 - s_1 sh^2 \alpha\phi_1 - s] \cdot \Delta_1^{-1}(\alpha), \quad (15)$$

$$H_{22}(\alpha) = i \frac{m_0 m_1}{\sqrt{n_1}} [s \cdot s_1 ch^2 \alpha\phi_1 + (\alpha\phi_1)^2 - \alpha\phi_1 \xi(\alpha) - s_1^2 sh^2(\alpha\phi_1) - s \cdot s_1] \cdot \Delta_1^{-1}(\alpha),$$

for $n_1 \neq n_2$

$$\widetilde{H}_{21}(\alpha) = m_0[-ss_1(\alpha\phi_1)\xi_2(\alpha) - s\xi_3(\alpha) + s(\alpha\phi_1)\xi_2(\alpha) + \xi_3(\alpha)] \cdot \Delta_2^{-1}(\alpha), \widetilde{H}_{22}(\alpha) =$$

$$= i \frac{m_0 m_1}{\sqrt{n_1}} [1 - s_1 ch(2\alpha\phi_2) + ss_1 \xi_1(\alpha) + s\alpha\phi_1 \xi_4(\alpha) + ss_1(\alpha\phi_1)^2 sh^2 \alpha\phi_1 -$$

$$- ss_1 ch^2 \alpha\phi_{21} - s_1^2(\alpha\phi_1)\xi_4(\alpha) + \xi_3(\alpha)] \cdot \Delta_2^{-1}(\alpha), \quad (16)$$

here n_1 and n_2 are the roots of the defining equation [1,13]. The quantities appearing in formulas (13), (14), (15), (16) are expressed in terms of the known parameters of the initial stress state [1,14].

Solving the system of recurrent equations. We apply the principle of superposition to determine the displacement of the points of the elastic band in the direction $0y_1$ and $0y_2$ under the simultaneous action of normal and tangential stresses for compressible and incompressible bodies for potentials of an arbitrary structure [1]

$$u_1(y_1) = \int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau + \int_{-\infty}^{\infty} h_{12}(|y_1-\tau|)q(\tau)d\tau, \quad (17)$$

$$u_2(y_1) = \int_{-\infty}^{\infty} h_{21}(|y_1-\tau|)p(\tau)d\tau + \int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q(\tau)d\tau.$$

Following [1,9], according to the system of equations

$$\frac{du_2(y_1)}{dy_1} = 0, \ (-\infty \prec y_1 \prec \infty) \quad (18)$$

$$E_1(y_1)\frac{du_1(y_1)}{dy_1} = \frac{1}{h}\int_{-\infty}^{y_1}[2q(t) - q_0(t)]dt.$$

Assuming that the heterogeneity of the stringer varies according to the law

$$E_1(y_1) = \mathrm{E}[(1+\delta f(y_1)], \ (-\infty \prec y_1 \prec \infty), \quad (19)$$

where $f(y_1)$ is known function, δ is small parameter.

Using the contact boundary conditions (9) and presenting the unknown contact stresses$p_0(y_1)$, $q_0(y_1)$ in the form of series of a small parameter

$$q_0(y_1) = \sum_{k=0}^{\infty} \delta^k q^{(k)}(y_1), \ (-\infty \prec y_1 \prec \infty), \quad (20)$$

let's write the system of solutions of recurrent systems of equations

$$\frac{du_2{}^{(0)}(y_1)}{d(y_1)} = 0, \ (-\infty \prec y_1 \prec \infty) \quad (21)$$

$$E_0 h\frac{d^2u_1{}^{(0)}(y_1)}{d(y_1)^2} = 2q^{(0)}(y_1) - q_0(y_1),$$

$$\frac{du_2{}^{(k)}(y_1)}{d(y_1)} = 0, \ (k=1,2,\ldots) \ (-\infty \prec y_1 \prec \infty), \quad (22)$$

$$E_0 h\frac{d^2u_1{}^{(k)}(y_1)}{d(y_1)^2} = 2q^{(k)}(y_1) - q_0{}^{(k-1)}(y_1),$$

where

$$q_0^{(k-1)}(y_1) = h\mathrm{E}_0\frac{d}{d(y_1)}\left[f(y_1)\frac{du_2{}^{(k-1)}(y_1)}{d(y_1)}\right], \ (k=1,2,\ldots),$$

$$u_1(y_1) = \int_{-\infty}^{\infty} h_{21}(y_1-\tau)p^{(k)}(\tau)d\tau + \int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q^{(k)}(\tau)d\tau,$$

$$u_2(y_1) = \int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p^{(k)}(\tau)d\tau + \int_{-\infty}^{\infty} h_{12}(y_1-\tau)q^{(k)}(\tau)d\tau, \ (-\infty \prec y_1 \prec \infty, k = 0,1,\ldots),$$

$$f_1^{(k-1)}(y_1) = D_0\frac{d^2}{d(y_1)^2}\left[f(y_1)\frac{d^2u_2{}^{(k-1)}(y_1)}{d(y_1)}\right], \ (k=1,2,\ldots), \quad (23)$$

$$f_2{}^{(k-1)}(y_1) = E_0 h\frac{d}{d(y_1)}\left[f(y_1)\frac{du_1^{(k-1)}(y_1)}{d(y_1)}\right], D_0 = E_0 I.$$

where D_0 is the zero term of the expansion for the series, $D(y_1) = IE_1(y_1)$ – bending stiffness of the stringer, I – parameter of inhomogeneity.

System (21) describes the contact problem for a homogeneous infinite stringer [6,13,15], each subsequent system with (22) differs from the previous external load. The solution of the contact problem for a prestressed strip with a heterogeneous infinite stringer is simplified to the solution of a series of uniform contact problems with different external loads system (21). The zero approximate solution, that is, the solution of system (21) using the Fourier transform, is constructed in [1] and has the form

$$\mathrm{p}(y_1) = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\left[\alpha^2\mathrm{H}_{21}^*(\alpha)\tilde{\mathrm{q}}_0(\alpha) + \mathrm{H}_{22}^*(\alpha)\tilde{\mathrm{p}}_0(\alpha)\right]\mathrm{H}^{-1}(\alpha)e^{-i\alpha y_1}d\alpha; \ (-\infty \prec y_1 \prec \infty) \quad (24)$$

$$\mathrm{q}(y_1) = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\left[\mathrm{H}_{11}^*(\alpha)\tilde{\mathrm{q}}_0(\alpha) - i\mathrm{H}_{12}^*(\alpha)\tilde{\mathrm{p}}_0(\alpha)\right]\mathrm{H}^{-1}(\alpha)e^{-i\alpha y_1}d\alpha.$$

Here, the quantities $H^{-1}(\alpha)$, $H_{ij}^*(\alpha)(i,j=1,2)$, are expressed through functions $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)(i,j=1,2)$, are determined by the formulas of equal and unequal roots of the equation [1,3,6,13] in the case of the specific structure of elastic potentials. The rest of the approximate solutions of influence of the heterogeneity of the stringer material are constructed in a similar way, where $\tilde{p}_0(\alpha)$ and $\tilde{q}_0(\alpha)$ is the Fourier, and μ is the Lamé coefficient.

Thus, the k approximation has the form

$$p^{(k)}(y_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P^{(k)}(s)\,e^{-isy_1}ds, \ q^{(k)}(y_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Q^{(k)}(s)\,e^{-isy_1}ds, \ (k=1,2,\ldots),$$

where

$$P^{(k)}(s)=\left.Ds^2\left\{\bar{f}_1^{(k-1)}(s)\left[E_0hs^2H_{22}(s)+1\right]-E_0hs^3\bar{f}_2^{(k-1)}(s)H_{12}(s)\right\}\right/L(s),$$

$$Q^{(k)}(s)=\left.-IE_0hs\left\{\bar{f}_2^{(k-1)}(s)\left[D_0hs^4H_{11}(s)+1\right]+D_0hs^3\bar{f}_1^{(k-1)}(s)H_{12}(s)\right\}\right/L(s), \quad (25)$$

$$(k=1,2,\dots),$$

are Fourier transforms of contact stresses.

In (25):

$$L(s)=[D_0s^4H_{11}(s)-1][E_0hs^2H_{22}(s)+1]+D_0E_0s^4hH_{12}^2(s),$$

$$\bar{f}_j^{(k-1)}(s)=F\left[f_j^{(k-1)}(y_1)\right],\ (j=1,2),\ (k=1,2,\dots),$$

where F is the Fourier transform operator for the specified function (functional).

Applying to (21) and (22) the integral Fourier transform to the contact stress transforms:

$$h_{11}(y_1)p^{(0)}(y_1)-Ih_{12}(y_1)Q^{(0)}(y_1)=0$$

$$E_0hy_1{}^2h_{21}(y_1)p^{(0)}(y_1)-[E_0hy_1{}^2h_{22}(y_1)+2]Q^{(0)}(y_1)=Q_0(y_1). \quad (26)$$

$$h_{11}(y_1)p^{(k)}(y_1)-Ih_{12}(y_1)Q^{(k)}(y_1)=0\ (k=1,2,\dots)$$

$$E_0hy_1{}^2h_{21}(y_1)p^{(k)}(y_1)-[E_0hy_1{}^2h_{22}(y_1)+2]Q^{(k)}(y_1)=Q_0{}^{(k-1)}(y_1) \quad (27)$$

where

$$\mathrm{p}^{(k)}(\mathrm{y}_1)=\mathrm{F}\left[\mathrm{p}^{(k)}(\mathrm{y}_1)\right],\ \mathrm{Q}^{(k)}(\mathrm{y}_1)=\mathrm{F}\left[\mathrm{q}^{(k)}(\mathrm{y}_1)\right]\ (\mathrm{k}=0,1,2,\dots).$$

$$Q_0(y_1)=F[q_0(y_1)],\ Q_0{}^{(k-1)}(y_1)=F\left[q_0{}^{(k-1)}(y_1)\right],$$

and I is the heterogeneity parameter, and $h_{ij}(y_1)$ is the influence function, the expressions:
for equal roots $n_1=n_2$:

$$h_{11}(y_1)=\frac{1}{\pi}\int_0^\infty H_{11}(\alpha)cos\alpha y_1\,d\alpha, \quad (28)$$

$$h_{12}(y_1)=\frac{1}{\pi}\int_0^\infty H_{12}(\alpha)sin\alpha y_1\,d\alpha,$$

for unequal roots $n_1\neq n_2$:

$$h_{11}(y_1)=\frac{1}{\pi}\int_0^\infty \widetilde{H}_{11}(\alpha)cos\alpha y_1\,d\alpha, \quad (29)$$

$$h_{12}(y_1)=\frac{1}{\pi}\int_0^\infty \widetilde{H}_{12}(\alpha)cos\alpha y_1\,d\alpha$$

Here, $h_{ij}(\alpha), i,j=1,2$ the influence functions, which characterize the movement of the limit points $y_2=0$ of an infinite elastic band with initial stresses from a unit horizontal force, core $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)$, respectively, have the form (15), (16).

After finding the transformants of the contact stresses from systems (26), (27) and applying the inverse Fourier transform, the expressions of the zero and k approximation of the normal and tangential stresses

$$p^{(0)}(y_1)=\frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{H^*{}_{12}(\alpha)}{H^*(\alpha)}Q_0(\alpha)sign\alpha e^{-i\alpha y_1}d\alpha;\ (-\infty\prec y_1\prec\infty) \quad (30)$$

$$q^{(0)}(y_1)=\frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{H^*{}_{11}(\alpha)}{H^*(\alpha)}Q_0(\alpha)e^{-i\alpha y_1}d\alpha;.$$

$$p^{(k)}(y_1)=\frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{H^*{}_{12}(\alpha)}{H^*(\alpha)}Q_0{}^{(k-1)}(\alpha)sign\alpha e^{-i\alpha y_1}d\alpha;\ (-\infty\prec y_1\prec\infty) \quad (31)$$

$$q^{(k)}(y_1)=\frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{H^*{}_{11}(\alpha)}{H^*(\alpha)}Q_0{}^{(k-1)}(\alpha)e^{-i\alpha y_1}d\alpha.$$

Here $H(\alpha)$, $H_{ij}^*(\alpha)(i,j=1,2)$ the quantities are expressed through known functions $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)(i,j=1,2)$, which are determined by the formulas of equal and unequal roots of the equation [1,16] for the corresponding structure of elastic potentials.

Expressions of contact stresses (30) describe the solution of the problem for a uniform stringer, which is the zero approximation of the problem for a non-homogeneous stringer. In this way, the approximations specified by formulas (31) create a variant of the solution for a non-homogeneous stringer. At the same time, it is possible to solve contact problems of an elastic body, with an infinite stringer, in which a weak heterogeneity varying according to the law is simulated

$$E_1(y_1)=\mathrm{E}[(1+\delta f(y_1)]\ (-\infty\prec y_1\prec\infty)$$

where $f(y_1)$ is known function, δ – small parameter.

System of solving equations. Having applied to both parts of the system (2.8) the integral Fourier transform by variable and using the convolution theorem, we find the expressions.

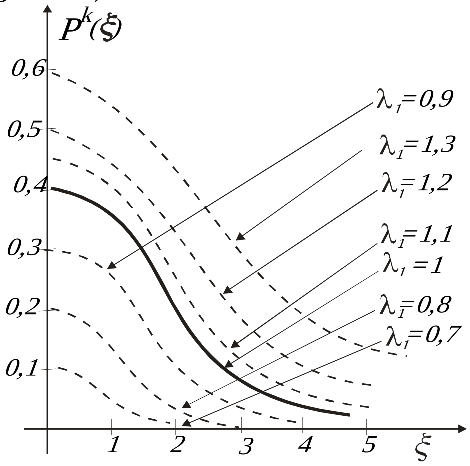
The zero approximation for the cases of equal and unequal roots of the characteristic equation will take the form (24), if these formulas are substituted:

- for equal roots $(n_1 = n_2)$ $H^*_{ij}(\alpha)$ on $H_{ij}(\alpha)$,

- for unequal roots on $(n_1 \neq n_2)$ $H^*_{ij}(\alpha)$ on $\widetilde{H}_{ij}(\alpha)$,

- where the kernels $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)$ have the form (1.11) and (1.12), respectively.

Let's consider numerical examples for incompressible bodies of neo-Hookian material (Treloir potential) (Fig. 2, a, b).



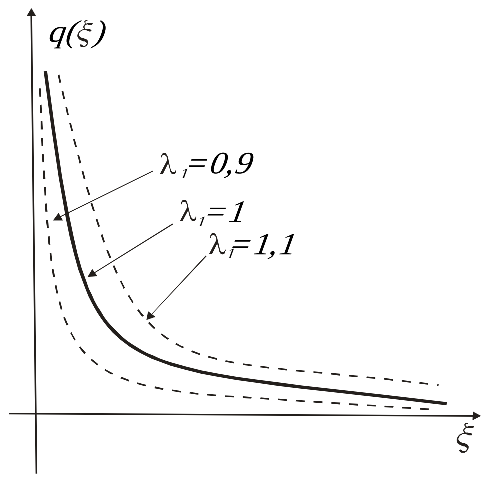


Fig. 2. **Numerical examples for incompressible bodies of neohouk material: a – for the dependence of $p^{(k)}(\xi)$ on ξ, b – for the dependence of q(ξ) on ξ**

Here $p(\xi), q(\xi)$ are dimensionless contact normal and tangential stresses in elastic strips with initial stresses. The value $\lambda_1 = 1$ corresponds to the results obtained in the work [11,16], $\lambda_1 = 0; 0{,}8; 0{,}4$ – corresponds to the initial compression stresses, and ξ – the initial tensile stresses, is a dimensionless coordinate of the initial stress state in the elastic band with the initial stresses.

The analysis of the graphs shows that in the case of compression $(\lambda_1 \prec 1)$, The presence of initial stresses in the strip leads to a significant reduction in tensile stresses $(\lambda_1 \succ 1)$ - to their increase.

Conclusions. In the work using the linearized theory of elasticity, it was possible to obtain a solution of the planar contact problem with the transfer of a concentrated horizontal load from an inhomogeneous infinite elastic stringer to prestressed strips with pinched faces. The calculations were based on the theory of large initial deformations and established several variants of small initial deformations. The solution of the problem is determined for normal and tangential contact stresses thanks to the system of recurrent integro-differential equations. The zero approximate solution of the inhomogeneous problem is built on the integral Fourier transform, which led to the representation of contact stresses by Fourier integrals.

1. The presented general case for equal and unequal roots of the equation of contact problems based on the linearized theory of elasticity makes it possible to formulate a general solution method that determines the solution of similar problems with known linear (without initial stresses) parameters.

2. For the case of equal roots of the defining equation for bodies with elastic potentials of arbitrary shape, the stresses and displacements at the ends of elastic pads have a feature that completely coincides with the feature in similar problems of the classical linear theory of elasticity. With unequal roots for bodies with elastic potentials of arbitrary shape, it is not possible to prove the coincidence of the orders of the specified features.

3. The contact stresses at the contact line with the elastic overlay are significantly dependent on the initial stresses. For highly elastic materials, the initial stresses play a more significant quantitative effect. Qualitative influence has an identical character.

References:

1. *Діхтярук М.М.* Періодична контактна задача для пружної смуги з початковими (залишковими) напруженнями. Доповіді національної академії наук.– 2004. № 3. – С. 46 – 49. 2. *Yuan*

W.K., Long J.M., Ding Y., Wang G.F. Micro/Nanocontact Between a Rigid Ellipsoid and an Elastic Substrate With Surface Tension // Journal of Applied Mechanics. – 2017. №1. – p. 011012, Paper No: JAM-16-1460. 3. *Stashchuk M.G., Nitrebych Z.M., Zhiguts Yu.Yu., Lazar V.F. at all.* Methods of calculating singular integrals for engineering problems // The 7th International scientific and practical conference "International scientific innovations in human life" (January 19-21, 2022) Cognum Publishing House, Manchester, United Kingdom. 2022. – P. 164-171. 4. *Zhiguts Yu., Lazar V., Pallay D.* Features of calculation of stresses caused in metal structures // The VIII International Scientific and Practical Conference «Actual trends in science and practice», February 28 – March 02, Geneva, Switzerland. 2022. – P. 162- 164. 5. *Akopyan V.N., Mirzoyan S.A., Mkhitaryan S.M.* The Problem of the Contact Between a Broken Stringer and an Elastic Infinite Strip Containing a Vertical Edge Crack // Int. Appl. Mech. – 2015. – 51, N 2. – P. 176 – 186. 6. *Babich S., Zhiguts Y., Lazar V.* System of recurrent equations for the problem of load transfer from an infinite inhomogeneous stringer to two jointed elastic strips // The 12th International scientific and practical conference "Actual issues of the development of science and ensuring the quality of education" (March 28 – 31, 2023) Florence, Italy. International Science Group. 2023. Pp. 347-353. 7. *Babich, S.Y., Yarets'ka, N.O.* Contact Problem for an Elastic Ring Punch and a Half-Space with Initial (Residual) Stresses. // International Applied Mechanics. 2021, №6. – p. 297-305. 8. *Moses O. P., Adewale A. O., Olusegun O. A.* Numerical Analysis of Thermo-Elastic Contact Problem of Disc Brakes for Vehicle on Gradient Surfaces. // World Journal of Engineering and Technology. – 2016. №1. – p. 51 – 58. 9. *Babich S.Yu., Zhiguts Yu.Yu., Lazar V.F.* Dynamic tensioned twolayer semispace under the influence of moving load // The 9th International scientific and practical conference "Modern science: innovations and prospects" (May 29-31, 2022) SSPG Publish, Stockholm, Sweden. 2022. – P. 201-207. 10. *Yaretskaya N.A.* The impact of the initial (residual) stresses on the contact interaction of elastic cylindrical punch and an elastic layer // Bulletin of NAS of Ukraine. – 2014. – N 1. – P. 57 – 62. 11. *Dikhtyaruk N.N.* Load transfer from the infinite stringer to the two jammed along one edge identical stripes with initial (residual) stresses // Scientific journal of the Ternopil national technical university. – 2016. N 3. – P. 51– 61. 12. *Babych S., Zhiguts Yu., Lazar V., Dihtyaruk M.* Contact problems for preliminary strips reinforced by thin elastic overlays stringers // The 4th International scientific and practical conference "Innovations and prospects of world science" (December 1-3, 2021) Perfect Publishing, Vancouver, Canada. 2021. – P. 220-224. 13. *Yaretskaya N.A.* Three-Dimensional Contact Problem for an Elastic Layer and a Cylindrical Punch with Prestresses // Int. Appl. Mech. – 2014. N 4. – P. 378 – 388. 14. *Rudnitsky V.B., Dikhtyaruk N.N.* Interaction Between an Infinite Stringer and Two Identical Prestressed Strips: Contact Problem. // International Applied Mechanics. – 2017. №2. – p 149–155. 15. *Babich, S.Y., Dikhtyaruk, N.N., Degtyar, S.V.* Contact Problem for Two Identical Strips Reinforced by Periodically Arranged Fasteners with Initial Stresses // International Applied Mechanics. – 2019. №6 – p. 629 – 635. 16. *Bosakov S.V.* Finite Integral Transform Method in Static Problems for Inhomogeneous Plates // Int. Appl. Mech. – 2014. – 50, N 6. –P. 651– 663.

Рецензент Рубіш Василь Михайлович, проф., док. фіз.-мат. наук, завідувач Ужгородської лабораторії матеріалів оптоелектроніки та фотоніки Інституту проблем реєстрації інформації НАН України