UDK 621.9.02 (07)
DOI 10.36910/10.36910/6775-2313-5352-2023-23-01
Shvets S.[1], Astakhov V.[2], Kolesnyk V.[1]
[1] Sumy State University, 2, Rymskogo-Korsakova, 40007, Sumy, Ukraine;
[2] EWIE Group of Companies, 1250 Beach Cr., 48176, Saline, MI, USA

PROFILING OF CUTTING TEETH OF A GEAR SHAPING HEAD

This paper investigates a possibility of the use of the mathematical model of the cutting edge profile developed for turning form cutter in the profiling the cutting edge of the tooth of a gear gushing head. The corresponding cutting tooth of the gashing head was carried out with Mathcad commercial software package accounting for the rake and clearance angles. The use of the same mathematical model increases the degree of automation of the design of various types of form cutting tools, which is the goal of many research work in the field of tool design and its manufacturing. In order to apply the mathematical model of the tuning form tool during the profiling of cutting teeth of a gear gashing head, arrays of coordinates of the points of the involute profile of the tooth are formed accordingly. This was carried out accounting on the difference in the motions of cutting edge of the tuning form tool and that of a gear gushing head. Moreover, the obtained results allow to conclude that the developed generalized mode can be used not only for great gushing tools but also for gear generating tools as gear shaper cutters.
Keywords: *turning form cutting, gear gushing head, mathematical model, involute profile, coordinates of the gear and tool tooth profile.*

Introduction. There are various effective methods of cutting gears [1]. The gear shaping head implements the gear gushing method. The use of this method implies that the profile of the cutting tooth of the tool in the normal cross-section is the same as that of the machined gear. However, the application of the rake and clearance angles on the tool tooth leads to the condition where the profile of the cutting tool should be made tool geometry specific. As a result, this profile can be not the same as that of the gear. Therefore, special calculation is used to achieve the required profile of the cutting tooth.

The major objective of modern manufacturing is achieving fast productivity. This objective is achieved through the use of high productive modern machines as well as implementation of Computer Aided Manufacturing (CAM) [2]. The latter requires accurate geometrical models of the cutting tools especially of complicated profiles. As such, the profiling procedure can be considered as innovative technology [3].

When one designs a gear gushing head, he or she can use the mathematical model of the geometry of form turning cutter with a zero inclinational angle. The unification of models allows to design various cutting tools efficiently [4 – 8].

Literature Review. It is known that the involute profile of a tooth is generated using the fundamentals of gear meshing [9, 10] so that an array of the coordinates of such a profile can be generated.

The coordinates of any point M of the involute tooth profile in the polar coordinate system (Fig. 1), the origin of which is placed in the center of the gear, are determined by the angle θ_M and the radius r_M [11 – 14] related as

$$\theta_M = \tan\left(\cos^{-1}\left(\frac{r_o}{r_M}\right)\right) - \cos^{-1}\left(\frac{r_o}{r_M}\right), \tag{1}$$

where $r_o = d_o/2$ is the radius of the base circle;
r_M is polar radius of point M.

The diameter of the base circle is correlated with the pitch diameter, d as $d_o = d \cdot \cos\alpha$. The pitch diameter is calculated as $d = mz$ (m is the module, z is the number of teeth), and α is the pressure angle. Normally, $\alpha = 20°$.

Due to the fact that the involute cannot be inside the base circle, the tooth profile is involute only outside this circle, and the part of the profile inside the base circle receives the necessary shape during the manufacturing process. The outer diameter of the gear $d_e = d + 2m$. Hence,

$$\frac{d_o}{2} \le r_M \le \frac{d_e}{2}. \quad (2)$$

By changing the value of r_M within the specified limits, the angle θ_M is determined using Eq. (1). This allows one to switch from the considered polar to a new Cartesian coordinate system, the origin of which is in the center of the gear (Fig. 1):

$$y = r_M \cdot \cos\theta_M, \quad x = r_M \cdot \sin\theta_M. \quad (3)$$

The objective of the work is to establish conditions for the use the geometrical model of a tuning form cutter in the profiling of a gear shaping head.

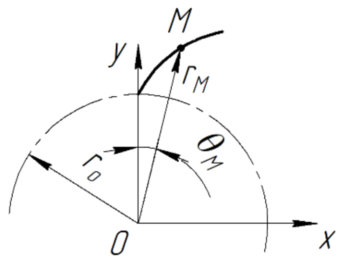


Figure 1– Coordinates of involute points in the polar coordinate system

Research Methodology. The current paper discusses the use of the mathematical model for turning form cutters of various types, proposed by S. V. Shvets [15], to calculate the profile of cutting teeth of a gear gashing head. Based on the theory of gear meshing, an array of coordinates of points of the involute profile of the gear tooth was generated. Using these generated data, the profiling of the corresponding cutting tooth of the gashing head was carried out with Mathcad commercial software package.

Results and Discussion. In order to use the mathematical model of a profiling cutter tooth, it is necessary to generate the output data accordingly.

For this purpose, the involute is moved over a circle so that it intersects with the arc of the base circle at a point located at the distance of a quarter of the pitch from the ordinate axis. The angle corresponding to one quarter of the gear pitch is calculated as

$$\delta = \frac{2\cdot\pi}{4\cdot z} = \frac{\pi}{2\cdot z}. \quad (4)$$

The involute angle on the pitch circle is calculated as

$$\theta_d = \tan\left(\cos^{-1}\left(\frac{r_o}{d/2}\right)\right) - \cos^{-1}\left(\frac{r_o}{d/2}\right), \quad (5)$$

The difference between them is

$$\mu = \theta_d - \delta. \quad (6)$$

The coordinates of the points of the involute profile R (Fig. 2) of the tooth placed at a distance of a quarter of an angular pitch from the y axis in the Cartesian coordinate system are determined as

$$x = r_M \cdot \sin(\theta_M + \mu), \quad y = r_M \cdot \cos(\theta_M + \mu). \quad (7)$$

Accordingly, the coordinates of the profile L of the opposite tooth (see Fig. 2) are determined as

$$-x = -r_M \cdot \sin(\theta_M + \mu), \quad y = r_M \cdot \cos(\theta_M + \mu). \quad (8)$$

The starting data for the calculation of the profile of the gear to be machined are arrays of coordinates of the points. Setting the value n, the number of points on the involute profile, one can determine the pitch between them as

$$\Delta = \frac{d_e - d_o}{2\cdot n}. \quad (9)$$

Array r_M is generated as

$$r_M = \left(\frac{d_e}{2} + \Delta, \frac{d_e}{2} + 2\Delta, \dots \frac{d_e}{2} + n\Delta\right). \tag{10}$$

The coordinate arrays of the points of the involute R in the xOy coordinate system are determined according to Eqs. (7), (1) and (10). Then, the transition into the x_a,O_a,y_a coordinate system (Fig 2) is made as follows

$$\begin{aligned} XR &= (x_1 + x_n, x_2 + x_n, \dots\ x_n + x_n), \\ YR &= (y_1 - y_n, y_2 - y_n, \dots\ y_n - y_n). \end{aligned} \tag{11}$$

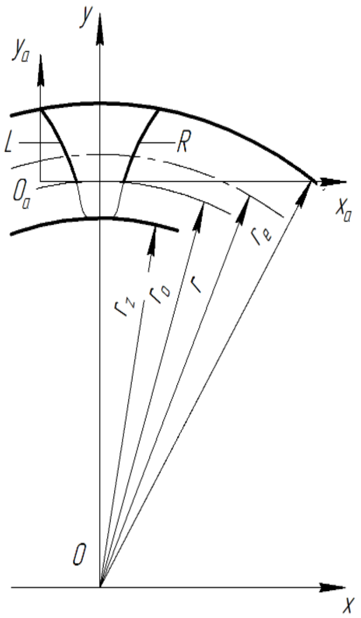


Figure 2– Coordinates of the involute profile of adjacent wheel teeth in the Cartesian coordinate systemFig. 1. Coordinates of involute points in the polar coordinate

Equations (8), (1) and (10) determine the arrays of coordinates of the points of the involute L in this system (Fig. 2)

$$\begin{aligned} XL &= (-x_1 + x_n, -x_2 + x_n, \dots\ -x_n + x_n), \\ YL &= (y_1 - y_n, y_2 - y_n, \dots\ y_n - y_n). \end{aligned} \tag{12}$$

Coordinates of 2n points of the involute profile of gear teeth in the $x_aO_ay_a$ coordinate system (Fig. 2) are:

$$\begin{aligned} XH &= (XL_n, XL_{n-1}, \dots\ XL_1,\ XR_1, \dots XR_n), \\ YH &= (YL_n,\ YL_{n-1}, \dots\ YL_1,\ YR_1, \dots YR_n). \end{aligned} \tag{13}$$

In addition to the specified arrays of coordinates, the initial data for calculations are the rake angle $\gamma = 5^o$ and the clearance angle $\alpha = 8^o$. However, depending on the work material, the rake angle can be as high as 25^o, and the clearance angle can be up to 12^o. The number of points on the profile is i = 1 ... 2n. The number of the point with the smallest YH value is n, and the number of the second point is n+1.

The difference between the motions of the tuning form tool and the cutting tooth of a gear gashing head is that the cutting edge of turning form cutter moves in a circle relative to the surface of the workpiece, whereas that of gear gushing head moves along a straight line relative to the workpiece. It is possible to convert a circle into a straight line by setting its radius to infinity. For engineering calculations, however, it is enough to increase each value of the YH array by 10^8. To achieve this, the outer radius of the round turning form cutter is set to be equal to 10^8, which allows us to consider this cutter as prismatic.

As a result of such a transformation, the mathematical model of the tooth profile of the gear gashing head, according to the profile of the prismatic turning form cutter with zero inclination angle [6], takes the form

$$\begin{gathered} F_i = -10^8 \tan\gamma, \\ y_i = \cos\gamma\left(\sqrt{YH_i^{\,2} - (F_i\cos\gamma)^2} - \sin\gamma F_i\right), \\ z_i = -y_i\tan\gamma - F_i, \\ YP_i = 10^8 - \sqrt{\begin{matrix}(z_i - 10^8\sin\alpha)^2 + \\ +(y_i - YH_n - 10^8\cos\alpha)^2\end{matrix}}. \end{gathered} \qquad (14)$$

Using the coordinates of the points of the involute profiles of the gear teeth (13), arrays *XH* and *YH*, one can determine the coordinates of the profile of a cutting tooth of a gear gashing head (*XH*, *YP*) using Eq. (14).

The mathematical model established by Eq. (14) can be used when profiling a gear cutter tooth. To form an array of coordinates of the points of the projection of the cutting edge of the tooth onto the reference plane of the gear cutter, the coordinate system x_bOy_b is used, the ordinate axis of which coincides with the axis of symmetry of the tooth and is rotated relative to the xOy coordinate system by the angle $\beta = \theta d + {}^{\pi}/_{2z}$ (Fig. 3). In this coordinate system, the coordinates of the points of the involute *I* are calculated as

$$\begin{cases} x_b = x\cos\beta - y\sin\beta \\ y_b = x\sin\beta + y\cos\beta \end{cases}. \qquad (15)$$

The coordinates along the abscissa axis of the opposite profile of the projection of the cutting edge *J* are defined as $-x_b$.

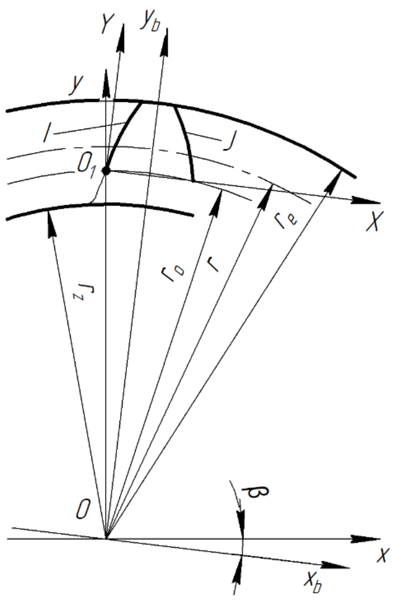


Figure 3– Determination of the coordinates of the projection points of the cutting edge on gear cutter face

Arras of coordinates of projection points *I* in the XO_1Y coordinate system are:

$$\begin{gathered} XI = (x_1 - x_1, x_2 - x_1, \dots\ x_n - x_1), \\ YI = (y_1 - y_1, y_2 - y_1, \dots\ y_n - y_1). \end{gathered} \qquad (16)$$

Arrays of coordinates of projection points *J* in the XO_1Y coordinate system are:

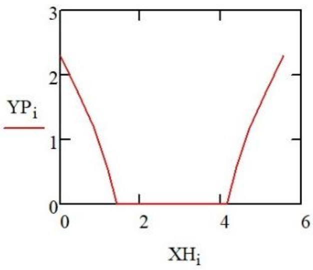
$$XJ = (-x_1 - x_1, -x_2 - x_1, \dots -x_n - x_1),$$
$$YJ = (y_1 - y_1, y_2 - y_1, \dots y_n - y_1). \quad (17)$$

Coordinate arrays of points of the entire projection of the cutting edge in the XO_1Y system:

$$XK = (XI_1, XI_2, \dots XI_n,\ XJ_n,\ XJ_{n-1}, \dots XJ_1),$$
$$YK = (YI_1,\ YI_2, \dots\ YI_n, YJ_n,\ YJ_{n-1}, \dots YJ_1). \quad (18)$$

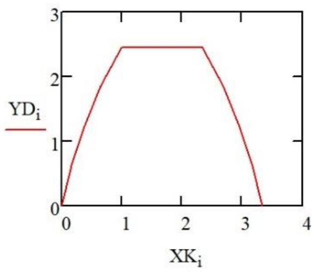
In addition to the specified arrays of coordinates, the rake angle γ = 5° and the clearance angle α = 6° are assigned. The number of points on the projection of the cutting edge i = 1...2n. The number

XH_i =	YP_i =
0	2.288545
0.442649	1.732507
0.829484	1.164625
1.155335	0.586641
1.410726	0
4.140422	0
4.395813	0.586641
4.721664	1.164625
5.108499	1.732507
5.551148	2.288545



a)

XK_i =	YD_i =
0	0
0.148	0.613
0.367	1.228
0.649	1.843
0.989	2.455
2.337	2.455
2.677	1.843
2.959	1.228
3.178	0.613
3.326	0



b)

Figure 4– Calculation of the coordinates of the points of the profile:
a) tooth of a gear gashing head, z = 19, m = 2, γ = 8°, α = 10°,
b) gear cutter tooth, z = 17, m = 2, γ = 5°, α = 6°

of the point with the smallest YK value is i = 1, and the number of the second point is 2n. Using the arrays (18) as initial data, according to the Eqs (14), the coordinates of the profile of the gear shaper tooth, the YD array. The XK array remains unchanged.

The coordinates of the points of the profile of a tooth of a gear gashing head and the tooth of the gear shaper are shown in Fig. 4.

Conclusions. On the basis of the theory of gears, arrays of coordinates of points of involute profiles of gear teeth were generated: XH – along the abscissa axis, YH – along the ordinate axis. The accuracy of the profile reproduction depends on the number of selected points n. The mathematical model of turning form tools was found to be suitable for profiling cutting tooth of a gear gashing head. This model can also be used when profiling the tooth of a gear shaper, during which the gear generating method is used. Thus, cutting edge profiles of various types of cutting tools are covered by one mathematical model, which increases the degree of automation of the tool design.

Information sources

1. Egorov, S., Kapitonov, A., Loktev, D. (2016). Manufacturing of cylindrical gears by generating cutting processes: A critical synthesis of analysis methods, *CIRP Annals*, Volume 870, pp. 331-342, https://doi.org/10.4028/www.scientific.net/MSF.870.397.

2. Bouzakis, K.-D., Lili, E., Michailidis, N., Friderikos, O. (2008). Manufacturing of cylindrical gears by generating cutting processes: A critical synthesis of analysis methods, *CIRP Annals*, Volume 57, Issue 2, pp. 676-696, https://doi.org/10.1016/j.cirp.2008.09.001.

3. Ziyong Ma, Yuanxin Luo, Yongqin Wang, Lian Mao (2018). Geometric design of the rolling tool for gear roll-forming process with axial-infeed, *Journal of Materials Processing Technology*, Volume 258, pp. 67-79, https://doi.org/10.1016/j.jmatprotec.2018.03.006.

4. Rodin, P. R. (1977). *Fundamentals of shaping surfaces by cutting*, Vyscha schkola, Kyiv, Ukraine.

5. Radzevich, S. P. (2001). *Shaping of surfaces of details. Fundamentals of the theory*, Rastan, Kyiv, Ukraine.

6. Matuszak, J., Barszcz, M. (2015). Computer aided design of cutting tools, *Advances in Science and Technology – Research Journal*, 9(28), pp.107-111, https://doi.org/10.12913/22998624/60796.

7. Bailey, J., Roy, R., Harris, R., Tanner, A. (2001). Cutting tool design knowledge capture, *Industrial knowledge management*, pp. 393-411, https://doi.org/10.1007/978-1-4471-0351-6_25.

8. Chung-Biau Tsay, Wen-Yao Liu, Yi-Cheng Chen (2000). Spur gear generation by shaper cutters, *Journal of Materials Processing Technology*, Volume 104, Issue 3, pp. 271-279, https://doi.org/10.1016/S0924-0136(00)00570-7.

9. Michalski, J., Skoczylas, L. (2008). A comparative analysis of the geometrical surface texture of a real and virtual model of a tooth flank of a cylindrical gear, *Journal of Materials Processing Technology*, Volume 204, Issue 1-3, pp. 331-342, https://doi.org/10.1016/j.jmatprotec.2007.11.038.

10. Zablonskii, K. I., Belokonev I. M., Shchekin B. M. (1989). *Theory of mechanisms and machines*, Vyscha schkola, Kyiv, Ukraine.

11. Härtig, F., Stein, M. (2019). 3D involute gear evaluation – Part 1: Workpiece coordinates, *Measurement*, 134, pp.569-573, https://doi.org/10.1016/j.measurement.2018.10.088.

12. Moath Zaareer (2021). A Novel approach for designing Gear tooth Profile by Mathematical equations for three different gear types Spur, Internal, and Helical Gear and creating models using SolidWorks and CATIA, *International Journal of Modern Science and Technology*, 6(2-3), pp. 31-42.

13. Suresh Babu, V., Majumder, M. C., Ramprasad, A. (2018). Involute Tooth Spacing, Gear Profile and 3D Gear development with MATLAB® Graphical User Interface and Solidworks, *International Journal of Industrial Engineering*, 2(3), pp. 67-75.

14. Shvets, S. V. (2019). *Metal cutting tools*, Sumy State University, Sumy, Ukraine.

Швець С.В.[1], Астахов В.П.[2], Колесник В.О.[1]
[1] Сумський державний університет, 2, Римського-Корсакова, 40007, Суми, Україна;
[2] EWIE Group of Companies, 1250 Beach Cr., 48176, Saline, MI, USA

ПРОФІЛЮВАННЯ РІЗЦІВ ЗУБОДОВБАЛЬНОЇ ГОЛОВКИ

У даній роботі досліджено можливість використання математичної моделі профілювання, розробленої для фасонних різців, при профілюванні різців зубодовбальної головки. Відповідне профілювання різця зубодовбальної головки було виконане за допомогою комерційного програмного пакету Mathcad з урахуванням переднього і заднього кутів. Використання однієї і тієї ж математичної моделі підвищує ступінь автоматизації проектування різних типів різальних інструментів, що є метою багатьох наукових робіт у галузі проектування та виготовлення інструменту. Для застосування математичної моделі профілювання фасонних різців під час профілювання різців зубодовбальної головки відповідно формують масиви координат точок евольвентного профілю зуба колеса. Це виконується з урахуванням відмінності рухів різальних кромок фасонного різця і різця зубодовбальної головки. Крім того, отримані результати дозволяють зробити висновок про те, що розроблений узагальнений метод може бути використаний не тільки для профілювання різців зубодовбальної головки , а й для зубоутворювальних інструментів, які використовують метод обкочування.

Ключові слова: *токарна обробка, зубодовбальна головка, математична модель, евольвентний профіль, координати профілю шестерні і зуба інструмента.*