УДК 656.11, 656.13
UDC 656.11, 656.13

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THE AVERAGE VEHICLE SPEED IN A DENSE TRAFFIC FLOW WITH THREE SLOW-MOVING VEHICLES ON A ROAD SECTION WITH LIMITED MANOEUVRING OPPORTUNITIES

Forecasting the urban traffic flow parameters remains one of the most challenging tasks for scientists and practitioners around the globe. The fundamental diagram of a traffic flow, which is a statistically established relationship between flow parameters, helps to solve this task. This diagram shows a decrease in speed with an increase in flow density, and one of the explanations for this is that drivers are forced to adapt to the speed of slower vehicles in front. This affects the variation of speed and indicates that it is reasonable to consider it a random variable with the mean and distribution, which depend on traffic conditions. For free flow, the normal vehicle speed distribution is generally recognised, while the issue of what distribution to use to represent the speed in dense traffic remains open. The current research contributes to the clarification of this issue, and its core is an elaboration of the process of forming an average speed of similar vehicles in the densest traffic flow on a road section with no overtaking opportunities. The search for this average value was started by considering the simplest situations of a vehicle platoon, where all vehicles are expected to move at the same maximum permitted speed except for one or two slow-moving vehicles that limit the speed of the others. This paper is a logical extension of these situations and is devoted to deriving a formula to calculate the average speed of a traffic flow with three slow-moving vehicles. A generalisation of the obtained results will create a sound basis for deriving a formula to calculate the average speed of a dense traffic flow with any number of slow-moving vehicles. The availability of this formula can be a starting point for understanding the changes in normally distributed free-flow speed caused by a dense traffic flow.

Keywords: traffic speed, average speed, traffic flow, flow density, traffic lane, traffic conditions, analytical modelling, transport system.

INTRODUCTION

Traffic flows (TFs) in modern cities significantly load street networks, causing many transport, environmental and safety problems [1]. Important information for their solution is the main TF parameters, including speed, which is particularly important [2]. The speed variable can be considered sufficiently studied in free-flow traffic conditions, under which it is well described by normal distribution [3]. This distribution is widely used in practice – for example, when planning traffic management measures, transport engineers and researchers estimate the percentiles of a normal distribution to set speed limits or advisory speeds, determine the modal and mean speed, pace speed, etc. [4].

The statement about the normal distribution of speed in free-flowing conditions is based only on empirical data. At the same time, it does not contradict the regularities in people's behaviour during personal and social activities [5-7], which undoubtedly include the satisfaction of mobility needs. On the other hand, the behaviour of road users in TF is determined by the individual characteristics of their mind, which results in the choice of a specific speed in certain traffic conditions. At the current level of knowledge, this excludes the possibility of substantiating the type of vehicle speed distribution analytically, making statistical tests of relevant hypotheses the main way to establish this distribution and estimate its parameters.

As for vehicle speed in complicated traffic conditions, no reference distribution, which scientists and practitioners would consider universal to represent it, has been established. Therefore, the issue of forecasting the TF parameters in urban areas, which are the places of concentration of the significant number of traffic problems, remains relevant. The fundamental diagram (FD) of TF, which is based on statistical data analysis, like the aforementioned normal speed distribution, helps to solve this issue. This diagram establishes a relationship between the flow speed, density and volume [2, 8, 9] and represents all TF states in one coordinate system. At the same time, the degree to which the FD considers the processes that occur in dense TFs, causing the variation of speed and affecting its average value, requires careful analysis.

LITERATURE REVIEW AND PROBLEM STATEMENT

The relationship between the main TF parameters has been keeping the attention of scientists and engineers for a long time. In 1930, American scientists led by Johnson A.N., while studying highway capacity, already noted a decrease in traffic speed with an increase in traffic volume and density [10]. In 1935, Greenshields B.D. published his work presenting a linear relationship between vehicle speed and traffic density, and his linear model is still popular [11].

The next in chronology is the TF study conducted in 1965 by Greenberg H., who presented a logarithmic relationship between speed and density [12]. His work also suggested that the speed depends only on the flow density, and TF behaviour can be described by analogy with classical fluid behaviour. At that time, the abovementioned suggestions could have helped represent the TF. Still, they cannot be considered fully justified because flow parameters depend on many factors – road surface and geometry, vehicle technical characteristics, driving style, driver's awareness of a particular road section, traffic conditions (lighting, weather, etc.) – that are not typical for the field of hydrodynamics [2, 13].

In 1965, the concept of discontinuous diagrams was established. The TF FD, which initiated this concept, was first demonstrated by the engineer and researcher Edie L.C. and represented the TF in the Holland Tunnel [14]. Edie L.C. showed that at the same traffic density, two different traffic volume values can exist on the diagram: a higher value for the flow entering the studied bottleneck and a lower value for the flow leaving it. The volume drop was explained by the difference in vehicle decelerations when approaching the bottleneck and accelerations after passing it.

Approximately in the same period, two conceptual works – the books by F.A. Haight [8] and D.R. Drew [9] – were published. They made a significant contribution to the development of the traffic flow theory. Slightly later, the book by H. Inose and T. Hamada [2] was published, and it also took a confident place in classical specialised literature.

In his book, F.A. Haight presents the FD plot, which is typical of modern research [8]. While categorically stating that any TF theory must correlate with the FD to be correct, the author also acknowledges that diagrams refer to specific places and traffic conditions and that there is controversial empirical data on the shape of the FD and the absence of a single function to represent it. F.A. Haight also attempts to find a speed distribution for the entire range of traffic conditions – from congested to free-flowing – and considers type I and type III distributions (beta and gamma families) for this purpose, taking into account that any driver cannot maintain the desired speed in the presence of other vehicles in the flow [8]. In these attempts, the author also notes that the problem of establishing a function to plot the FD can be reduced to the problem of determining the average vehicle speed as a function of the flow density. Using the mentioned distributions in the study of the movement of two vehicles – a leader and a car-follower that does not have the opportunity to overtake – the book's author tries to derive a function to represent the FD. In addition, the book considers the movements on a two-lane road with two-way traffic and overtaking opportunities to obtain the distribution of spot speeds; here, the waiting for the opportunity to overtake is perceived as a restriction on the movement, which leads to the formation of 'moving queues' [8]. These issues are studied using the queuing theory. However, using this theory raises some doubts as it is more appropriate for studying traffic controlled by traffic signals than for traffic without automatic control, where the flow parameters depend on the individual characteristics of drivers and should be determined empirically [8]. At the same time, it is worth noting that F.A. Haight's considerations on the possibility of maintaining the desired speed in the flow and the vehicle movement with no overtaking opportunities are relevant to the present and are pretty close to those that will be used in the problem statement in the current paper.

D.R. Drew's book points out that field surveys and the statistical analysis of their results are the traditional way to obtain traffic data, which is equally valid for the present time [9]. The author also emphasises the importance of speed as the traffic smoothness parameter and presents a more generalised measure for TF quality based on the average speed on a road section. The importance of the average speed value is also underlined when showing the prevalence of the use of the normal distribution to describe the speed variable [9].

In the book by H. Inose and T. Hamada, speed and other urban TF parameters are considered averaged characteristics of vehicle traffic, which is formed from vehicles having different technical specifications and randomly meeting on the network [2]. To estimate these parameters, the authors use the analogy of the TF to an incompressible fluid flow. In addition to the averaged vehicle speed presented on the FD, the book shows the derivation of the traffic speed distribution function as a weighted sum of the exponentially distributed time headway of vehicles in a free flow and the gamma-distributed (or Erlang-distributed) time headway in a dense flow [2]. The reason for using the latter distribution instead of the commonly used exponential distribution is that in dense traffic, the headways very close to zero cannot occur with high probability, as implied by the exponential distribution, because each vehicle has a non-zero length. F.A. Haight and D.R. Drew used the same headway distributions, but they did not use them to obtain the speed distribution. It is worth noting that H. Inose and T. Hamada, when considering the relationship between speed and density, assume that with increasing traffic density, drivers are forced to reduce their speed to maintain a safe distance and ensure traffic safety [2].

The research on TF FD became more extensive in the late XX and early XXI centuries. This period showed the development of existing and new approaches to describe the scattering of data points on volume-density, speed-density, and speed-volume plots [13]. During this time, S. Smulders published an approach [15, 16], according to which the part of the speed-density graph corresponding to the free flow was plotted as a straight line while the other part showed nonlinear dependence. However, the reasons for choosing the functions that produce such a graph were not explained.

Sometime later, a work that can be considered a development of the concept of discontinuous FDs was published [17]. This work presented a diagram based on a microscopic car-following model that differentiates driver behaviour in free and congested traffic flow. The advantage of the model used is taking into account the platooning process, albeit in a simplified way, and the disadvantages are as follows: 1) the use of a constant time headway for all traffic states up to the dense flow; 2) the possibility of determining one of the model parameters only based on the assumption that the maximum speed of a dense flow is equal to the speed of the free flow with total platooning.

More recent approaches to representing the FD include a highly flexible function proposed by J.M. del Castillo. The parameters of this function can be freely adjusted to obtain almost any desired graph to fit empirical data [18, 19].

Throughout the history of research on the FD, many models have been developed to represent the relationship between TF parameters at specific locations and to study driver behaviour in certain traffic conditions [20-23]. These models are so numerous that it is almost impossible to analyse all of them. However, there is no critical need for this since all models constitute an approximation of empirical data using the least squares method [20]. All mentioned models and the approaches to develop them do not question the existence of a regular relationship between TF parameters. At the same time, none of the models explains the reasons for the undeniable regularities reflected in the scattering of data points in the FD plots, including the ‘speed-density’ plot. The speed-density relationship indicates a decrease in the flow speed with an increase in density. In the literature, this is explained by the fact that as the traffic becomes saturated (dense), drivers reduce their speed to ensure traffic safety, i.e., to maintain a safe distance [2]. However, the distance between vehicles is determined not only by safety reasons but also by the fact that drivers are forced to adapt their speed to the speed of the vehicles in front, and this fact is confirmed in up-to-date research [13].

This adaptation certainly affects the variation of speed and its average value. From this point of view, the speed in a dense TF, which is represented by the rightmost part of the speed-density diagram, can reasonably be considered a random variable. Such consideration is the subject of many studies [24]. A part of them indicates a normal speed distribution [24], and the other part shows the possibility of using distributions with a left-shifted mode (with right asymmetry), namely lognormal, gamma, Weibull and similar [25, 26]. At the same time, the reasons for the change in speed values as the free-flow conditions change to complicated ones are either not explained or reduced to the heterogeneity of the traffic composition [26]. The latter explanation cannot be considered sufficient because the vehicle speed in a dense TF is not always explicitly determined by the vehicle type but formed by many random factors, the contribution of which is quite challenging to assess.

Therefore, a situation arises when the distribution of speed values located in the leftmost part of the speed-density diagram is known – it is a normal distribution, and the distribution of speeds concentrated in the rightmost part of the diagram is somewhat uncertain. To study the latter distribution, it is reasonable to examine the process of forming the average vehicle speed value, and the great attention to this parameter in the analysed literature supports this way of study. It is rational to develop such a study by considering practically possible situations of vehicle movement in dense traffic and a logical description of the processes that occur in the flow.

AIM AND TASKS

One of the reasons for the increase in traffic density on road network sections is that there are no alternative trajectories for drivers to pass those sections without interruptions. The change of TF state from free to dense can be reasoned by various situations, which are often the result of permanent or temporary traffic management measures or specific traffic conditions at certain road sections. These situations frequently occur in urban areas when a carriageway is narrowed to a single lane for heavy traffic and when vehicles leaving a signalised intersection enter a road section with only one lane available for traffic. These situations can also be caused by curb parking. In both mentioned situations, drivers are limited in their manoeuvrability and forced to follow slow-moving vehicles (SVs) until they can overtake these SVs travelling at a speed lower than the desired speed for others. Travelling at the speed limited by the vehicle in front divides the vehicle platoon into smaller platoons led by SVs. This creates the conditions for formalising the processes in the flow and deriving the dependence of the average flow speed on the speed of individual traffic participants under their strong influence on each other.

Both of the situations mentioned above – a carriageway narrowed to one lane and a signalised entry to a road section with only one lane available for traffic – are similar in terms of traffic conditions [24] since, quite often, vehicles start at a particular place from zero speed (from rest), enter a one-lane section and then form a platoon. The vehicles in this platoon can reach the desired speed or the speed prescribed by traffic rules. In the case of a signalised entrance to a road section, the start from rest is obvious, and the same conditions also arise in the case of a narrowed carriageway since the vehicles can only hypothetically enter such a carriageway without any stops [27]. Therefore, the condition of starting from rest and entering a one-lane section stipulates the need to estimate the average platoon speed at the exit from the section. To be correctly stated, the problem of determining the average speed in the listed situations should be considered for the one-lane sections having a length sufficient for the acceleration of the vehicles up to the desired speed (close or equal to the permitted speed) in traffic conditions which are free-flowing except for no overtaking opportunities. In other words, the average speed of the vehicles should be estimated at such a distance from the place of platoon formation that would allow all vehicles, which are not retarded by slow-moving traffic participants in the platoon, to attain the desired speed.

This paper deals with the task of determining the flow speed under conditions of strong interaction between traffic participants, which suggests obtaining the formula to calculate the average speed of the vehicles of the same type that would move at different speeds in free-flow conditions at the exit from a one-lane road section where overtaking is impossible. The search for this formula was started in papers [28] and [29], which considered the simplest situations of a dense platoon, in which all drivers want to travel at the same maximum permitted speed, except for (i) one slow-moving vehicle (SV) and (ii) two SVs that limit the speed of the others. The current paper is a logical continuation of this search. It is devoted to deriving the formula for the average vehicle speed in the case of three SVs in a platoon.

RESEARCH RESULTS

The presence of not one or two but three SVs in a TF is the next difficult task of estimating the average vehicle speed at the exit from a lane with no overtaking opportunities. To address this task, it is reasonable to formalise it in the following way:

- let N vehicles, the number of which is more than three ($N > 3$) and which drive in a dense flow, randomly enter a road section with one lane and no overtaking opportunities. Let the numbering of vehicle positions in the platoon formed at the section starts from 0. Then, under equal probability for each vehicle to take any position x in the platoon, each vehicle can have number $[0; N-1]$ (i.e., $x \in [0; N-1]$) with probability $1/N$;

- let all but three drivers want to drive at the maximum permitted speed $V_{\max}$;

- let the driver of the first SV (SV1) travels at speed V_s, $0 < V_s < V_{\max}$, the driver of the second SV (SV2) travels at speed V_2, $V_s < V_2 < V_{\max}$, and the driver of the third SV (SV3) travels at speed V_3, $V_2 < V_3 < V_{\max}$;

- let $\Delta_s = (V_{\max} - V_s)$ be the deviation of SV1's speed from the maximum permitted speed, $\Delta_2 = (V_2 - V_s)$ be the deviation of SV2's speed from the SV1's speed (the speed of the slowest vehicle), and $\Delta_3 = (V_3 - V_s)$ be the deviation of SV3's speed from the SV1's speed.

Before proceeding to the derivation of the formula for the average speed of the flow with three SVs at the exit from a lane with no overtaking opportunities, it is necessary to briefly summarise the results of considering a flow with one and two SVs presented in papers [28] and [29], respectively. In the presence of only one SV in the TF, the average vehicle speed under equal probability of SV to take any position x in the platoon was determined as

$$\bar{V} = \frac{1}{N}\sum_{x=0}^{N-1}\bar{V}_x = \frac{1}{N^2}\sum_{x=0}^{N-1}\left(x \cdot V_{\max} + (N-x)\cdot V_s\right), \tag{1}$$

where x is the in-flow position of the single SV which drives at speed V_s;

$\bar{V}_x$ is the average speed of the vehicles in the flow when the single SV takes position x with probability $1/N$;

$[x \cdot V_{\max} + (N - x) \cdot V_s]$ is the sum of the speeds of all vehicles in the flow when the single SV takes position x.

After mathematical transformations made in paper [28] taking into account designation for Δ_s, the following formula was obtained to calculate the average speed of the flow with a single SV on a lane with no overtaking opportunities:

$$\bar{V} = V_s + \frac{N-1}{2N}\Delta_s. \quad (2)$$

Based on this, the case with two SVs in the flow was considered in paper [29]. It was taken into account that SV2, having the speed V_2, may take position x ahead of SV1 with probability $1/N$. For the case of two SVs in the flow, it was found that for $(N - x - 1)$ vehicles behind position x in the TF, the above situation with one SV occurs, but with a maximum speed equal to V_2. The equation for the average speed of these vehicles $\bar{V}_x$, where x denotes the SV2's position in the flow, was derived taking into account Eq. (2) and the number of vehicles behind the position x [29]:

$$\bar{V}_x = V_s + \frac{N-x-2}{2(N-x-1)}\Delta_2. \quad (3)$$

Then, by analogy with Eq. (1), the following equation for the average flow speed, which considers all possible positions of two SVs among N vehicles, was written:

$$\bar{V} = \frac{1}{2}\bar{V}_1 + \frac{1}{2}\bar{V}_2, \quad (4)$$

where the terms represent the average vehicle speed which takes into account the possibility of a SV to take the position $x \in [0; N-2]$ in the flow (the presence of two SVs reduces the number of possible positions for each of them to $(N-2)$) – $\bar{V}_1$ is the average speed if SV1 takes the position x, and $\bar{V}_2$ is if the SV2 [29].

After determining the total number of situations when SV1 can take the position ahead of SV2, the formula for $\bar{V}_1$ for the case of two SVs in the flow was obtained: $\bar{V}_1 = V_s + \frac{\Delta_s}{N} \cdot \frac{N-2}{3}$. Then, using Eq. (3), the sum of speeds of all vehicles in the flow when SV2 takes position x ahead of SV1 was written down:

$$x \cdot (V_s + \Delta_s) + (V_s + \Delta_2) + (N - x - 1) \cdot \bar{V}_x. \quad (5)$$

Based on this sum, the formula for $\bar{V}_2$ for the case of two SVs in the flow was derived: $\bar{V}_2 = \bar{V}_1 + \frac{\Delta_2}{N} \cdot \left(1 + \frac{N-2}{3}\right)$ [29].

Substituting the obtained equations for $\bar{V}_1$ and $\bar{V}_2$ into Eq. (4) allowed for deriving the formula for the average speed of a dense flow with two SVs at the exit from a single-lane road section with no overtaking opportunities [29]:

$$\bar{V} = V_s + \frac{\Delta_s}{N} \cdot \frac{N-2}{3} + \frac{\Delta_2}{2 \cdot N} \cdot \frac{N+1}{3}. \quad (6)$$

The briefly presented results of previous studies of simpler cases with a smaller number of SVs in the flow [28, 29] are the necessary material to proceed to the subject of this paper, namely the consideration of the case with three SVs in the TF.

Since overtaking on the considered road section is impossible, in the case of three SVs, the speed of $(N - 3)$ vehicles, which want to travel at speed $V_{\max}$, depends on which SV they follow. If they follow the

SV1, they will exit the one-lane road section at speed V_s; if they follow SV2, they will exit at speed V_2; if they follow SV3, they will exit at speed V_3; and vehicles ahead of all the SVs will exit at speed $V_{\max}$. Obviously, the flow speed depends on the order of vehicles in it and the total number of possible vehicle orders in the flow is $N!$. According to the task stated, it is necessary to determine the average speed of vehicles at the exit from a one-lane road section with no overtaking opportunities, which considers all possible vehicle permutations in the flow.

The probability of the event that one of the three SVs takes the position x in the flow under a single realisation of the random order of vehicles is equal to $3/N$, and the probability that each of the three SVs takes the position x is the same for all SVs and equals to $1/N$. Under an equal probability of all vehicle orders in the flow, each vehicle will take the position x exactly $(N-1)!$ times, i.e., the variants for positioning of SVs are invariant with respect to the SV's number. However, the speed of the vehicle flow on the lane with no overtaking opportunities is completely determined by the order of vehicles, and to calculate the average speed for the entire flow, it is essential to take into account not so much the position of one of the three SVs as its location relative to other SVs. To consider this aspect, it is reasonable to divide the entire set of vehicle orders in the flow into subsets in which one of the SVs drives ahead of the other SVs. Since all orders are invariant with respect to the SVs' numbers, the quantity of situations in which SV1 (the slowest vehicle) is ahead of the other SVs is equal to the number of situations in which SV2 or SV3 is ahead. Therefore, by analogy with Eq. (1) and concerning Eq. (4) and the introduced designations, it is possible to write down the equation for the average flow speed, which considers all possible variants of the relative position of three SVs among N vehicles:

$$\bar{V} = \frac{1}{3}\bar{V}_1 + \frac{1}{3}\bar{V}_2 + \frac{1}{3}\bar{V}_3, \tag{7}$$

where $\bar{V}_1, \bar{V}_2, \bar{V}_3$ represent the average flow speed when SV1, SV2 or SV3 is ahead of the other SVs, respectively.

If SV1 travelling at speed V_s takes position x in the flow ahead of SV2 and SV3 (Fig. 1), the average flow speed at the exit from a lane will be the same as in the case of a single SV in the TF. It can be determined using Eq. (2). The number of these situations in the full set of vehicle permutations is equal to $N!/3$, and the probability of this event is equal to $1/3$. At that, the number of positions available for SV1 is reduced to the first $(N-2)$ positions in the platoon since SV2 and SV3 must necessarily be behind SV1.

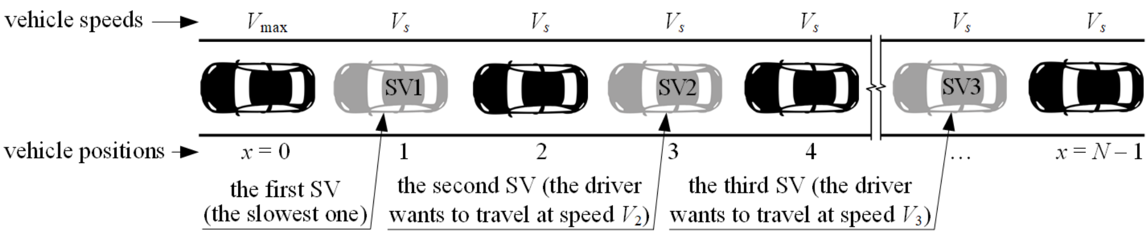

Figure 1 – Traffic flow with three SVs when the slowest one takes a position ahead of the other SVs

With the same probability $1/3$, both SV2, which wants to drive at speed V_2, and SV3, which wants to drive at speed V_3, can take position x ahead of the SV1.

If SV2 is ahead of the other SVs, then all vehicles ahead of it will drive at the maximum permitted speed $V_{\max}$, SV2 will drive at speed V_2, and the vehicles behind it will either drive at speed V_2 if there is no SV1 ahead of them (Fig. 2a) or at minimum speed V_s otherwise (Fig. 2b). A similar situation for a full set of N vehicles with the same speeds has already been studied in paper [29] and represented by Eq. (6), but in the situation under consideration in the current paper – when there are three SVs in the flow – the sum limit will depend on the position of SV2 in the flow.

If SV3, travelling at speed V_3, will take the position x in the flow ahead of the two slowest SVs, then all vehicles ahead of it will drive at maximum permitted speed $V_{\max}$, SV3 will drive at speed V_3, and the vehicles behind it will drive at speed V_3 if there are no two slowest SVs ahead of them. If, however, one of the two slowest SVs is ahead of them, they will either drive at speed V_2 if SV2 travels in the tail of the platoon behind SV3 and ahead of the SV1 (Fig. 3a) or at speed V_s if in the tail of the platoon (behind SV3) SV1 travels ahead of the SV2 (Fig. 3b).

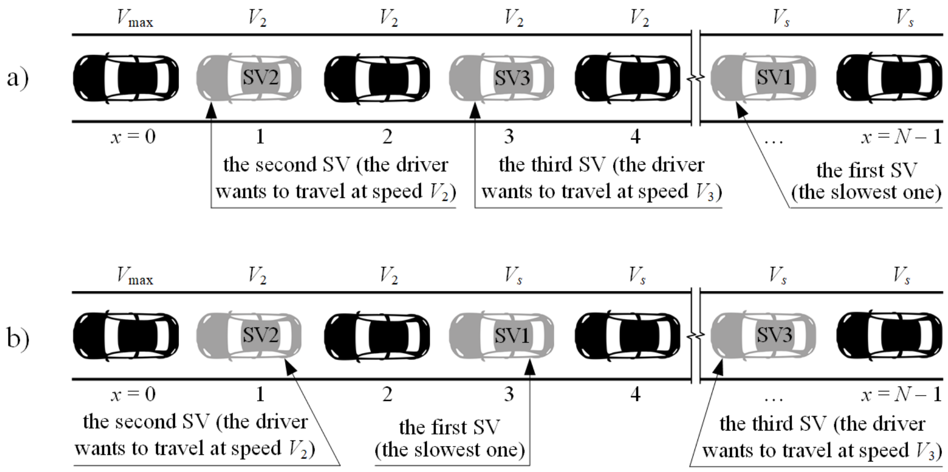

Figure 2 – Traffic flow with three SVs when SV2 takes the position ahead of the other SVs: a) the fastest SV is ahead of the slowest one; b) the slowest SV is ahead of the fastest one

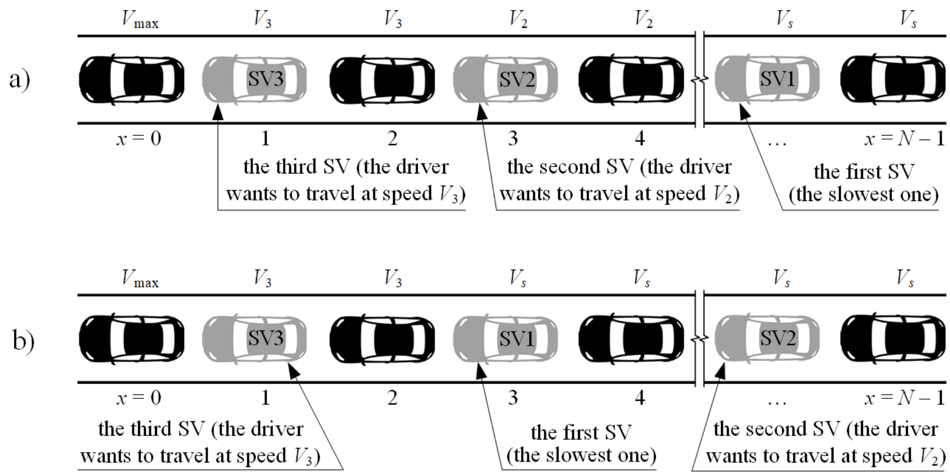

Figure 3 – Traffic flow with three SVs when the fastest one takes the position ahead of the other SVs:
a) SV2 is in the tail of the platoon and ahead of the slowest SV;
b) the slowest SV is in the tail of the platoon and ahead of SV2

Now, the average flow speed under different SV positions in the TF presented in Figs. 1-3 should be considered in detail. It is logical to start this process by deducing the analytic dependence for $\overline{V_1}$ in Eq. (7). At the beginning, it is appropriate to mention that the mandatory presence of two other SVs behind SV1, whether even slower or potentially faster, means that not all of N, but only first $(N-2)$ positions in the flow are available to SV1. Taking this into account, for each x-th position of SV1 in the flow, it is necessary to determine the number of situations when it takes that position, given that the other two SVs follow it. The total number of situations when SV1 can take each position in the flow without the condition of the mandatory presence of two other SVs behind it is known and is equal to

$$\frac{N!}{N} = (N-1)!, \tag{8}$$

i.e., the number of permutations of all other vehicles – fast and slow – regardless of the value of x. The dependence of the sought number of situations on the position of SV1 arises precisely from the condition of its location ahead of the other SVs.

In the first position ($x=0$), this condition is met automatically, and the number of situations when SV1 can take this position is always equal to Eq. (8).

As for the general case of taking the x-th position by SV1, it should be considered that the number of positions in the flow available to two other SVs following the SV1 is equal to $Q=(N-1-x)$. The number of variants for placing these two SVs on those available positions is equal to [30, 31]

$$A(Q,2)=\frac{Q!}{(Q-2)!}=Q\cdot(Q-1)=(N-1-x)\cdot(N-2-x).$$

At that, the fast vehicles (FVs) can take $(N-3)$ remaining positions, and the total number of variants for placing them in the TF equals $(N-3)!$. Consequently, according to the combinatorial rule of product [30, 31], the total number of situations when SV1 can take the position ahead of the other SVs is determined by the expression $(N-1-x)\cdot(N-2-x)\cdot(N-3)!$. The correctness of this expression is confirmed by the fact that under $x=0$, it transforms to the right side of Eq. (8). Also, under $x=(N-2)$ or $x=(N-1)$, the obtained expression results in 0, which is logical since in this case, there will be no available positions for placing two other SVs.

Using these findings, the following equation for $\bar{V}_1$ can be written down:

$$\bar{V}_1=\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-3)!\cdot\frac{(x\cdot[V_s+\Delta_s]+[N-x]\cdot V_s)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-3)!},$$

In the numerator and denominator of this formula, it is reasonable to take the multipliers $(N-3)!$, which do not depend on the summation index, out beyond the summation sign and cancel them. After this cancellation, the presented formula will look as follows:

$$\bar{V}_1=\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{(x\cdot[V_s+\Delta_s]+[N-x]\cdot V_s)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},\qquad(9)$$

where $(N-2-x)\cdot(N-1-x)$ can be considered the component that regards the number of situations when SV1 in a flow of N vehicles occurs in the position x and ahead of the other two SVs.

Eq. (9) represents the average speed of all vehicles in the flow, weighted by the number of situations when SV1 takes the position ahead of the other SVs. After the collection of like terms, Eq. (9) can be simplified to

$$\bar{V}_1=\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{(x\cdot\Delta_s+N\cdot V_s)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}.$$

Dividing the parenthetical expression $(x\cdot\Delta_s+N\cdot V_s)$ on the right of the numerator into two separate terms – $x\cdot\Delta_s$ and $N\cdot V_s$ – allows for taking the speed V_s and the multiplier at x out beyond the summation sign since the weighted average of the constant is equal to the constant itself and the mentioned multiplier does not depend on the summation index:

$$\bar{V}_1 = V_s + \frac{\Delta_s}{N} \cdot \frac{\sum_{x=0}^{N-3} (N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3} (N-2-x)\cdot(N-1-x)}. \tag{10}$$

The multiplier at $\frac{\Delta_s}{N}$ can be simplified by removing the brackets in its numerator and denominator and applying the formulas for the sum of series $\sum_{x=1}^{m} x = \frac{m\cdot(m+1)}{2}$, $\sum_{x=1}^{m} x^2 = \frac{m\cdot(m+1)\cdot(2\cdot m+1)}{6}$ and $\sum_{x=1}^{m} x^3 = \left(\frac{m\cdot(m+1)}{2}\right)^2$ [30]. Putting a zero term under the summation sign in these formulas, which will not change the sum totals since this term will be equal to zero, and substituting $m=(N-3)$ to bring the sum limit into compliance with the one used in Eq. (10) allows for obtaining the following:

$$\sum_{x=0}^{N-3} x = \frac{(N-3)\cdot(N-2)}{2}, \tag{11}$$

$$\sum_{x=0}^{N-3} x^2 = \frac{(N-3)\cdot(N-2)\cdot(2\cdot N-5)}{6}, \tag{12}$$

$$\sum_{x=0}^{N-3} x^3 = \left(\frac{(N-3)\cdot(N-2)}{2}\right)^2 = \frac{(N-3)^2\cdot(N-2)^2}{4}. \tag{13}$$

Removing the brackets in the numerator of the multiplier at $\frac{\Delta_s}{N}$ in Eq. (10) and collecting like terms results in

$$\begin{aligned}\sum_{x=0}^{N-3} (N-2-x)\cdot(N-1-x)\cdot x &= \sum_{x=0}^{N-3} x\cdot\left[(N^2-3\cdot N+2) - x\cdot(2\cdot N-3) + x^2\right] = \\ &= (N^2-3\cdot N+2)\cdot\sum_{x=0}^{N-3} x - (2\cdot N-3)\cdot\sum_{x=0}^{N-3} x^2 + \sum_{x=0}^{N-3} x^3.\end{aligned} \tag{14}$$

Substituting Eqs. (11)-(13) into Eq. (14) allows for simplifying this numerator as follows

$$\begin{aligned}&(N^2-3\cdot N+2)\cdot\frac{(N-3)\cdot(N-2)}{2} - (2\cdot N-3)\cdot\frac{(N-3)\cdot(N-2)\cdot(2\cdot N-5)}{6} + \frac{(N-3)^2\cdot(N-2)^2}{4} = \\ &= \frac{N\cdot(N^3-6\cdot N^2+11\cdot N-6)}{12} = \frac{N\cdot(N-1)\cdot(N-2)\cdot(N-3)}{12}.\end{aligned} \tag{15}$$

Removing the brackets in the denominator of the multiplier at $\frac{\Delta_s}{N}$ in Eq. (10) and collecting like terms results in

$$\begin{aligned}\sum_{x=0}^{N-3} (N-2-x)\cdot(N-1-x) &= \sum_{x=0}^{N-3}\left[(N^2-3\cdot N+2) - x\cdot(2\cdot N-3) + x^2\right] = \\ &= (N^2-3\cdot N+2)\cdot\sum_{x=0}^{N-3} 1 - (2\cdot N-3)\cdot\sum_{x=0}^{N-3} x + \sum_{x=0}^{N-3} x^2.\end{aligned} \tag{16}$$

Substituting Eqs. (11), (12) and $\sum_{x=0}^{N-3} 1 = 1 \cdot [(N-3) - 0 + 1] = (N-2)$ [30] into Eq. (16) allows for simplifying this denominator as follows

$$(N^2 - 3 \cdot N + 2) \cdot (N-2) - (2 \cdot N - 3) \cdot \frac{(N-3) \cdot (N-2)}{2} + \frac{(N-3) \cdot (N-2) \cdot (2 \cdot N - 5)}{6} = \\ = \frac{N \cdot (N^2 - 3 \cdot N + 2)}{3} = \frac{N \cdot (N-1) \cdot (N-2)}{3}. \quad (17)$$

The ratio of the numerator represented by Eq. (15) to the denominator represented by Eq. (17) results in the final expression for the multiplier at $\frac{\Delta_s}{N}$ in Eq. (10), which can be simplified as follows:

$$\frac{\sum_{x=0}^{N-3} (N-2-x) \cdot (N-1-x) \cdot x}{\sum_{x=0}^{N-3} (N-2-x) \cdot (N-1-x)} = \frac{N \cdot (N-1) \cdot (N-2) \cdot (N-3)}{12} \cdot \frac{3}{N \cdot (N-1) \cdot (N-2)} = \frac{N-3}{4}. \quad (18)$$

Taking into account Eq. (18), the average flow speed when SV1 is ahead of the other two SVs in the TF can be finalised as follows:

$$\bar{V}_1 = V_s + \frac{\Delta_s}{N} \cdot \frac{N-3}{4}. \quad (19)$$

The derivation of the formula for the second term in Eq. (7) is similar to the case of two SVs in a TF (Eq. (6)) since the presence of another vehicle willing to travel at a speed higher than that of SV2 does not cause any changes because that another vehicle will follow the slower vehicle and will be forced to adapt to latter's speed just as all the FVs in the flow behind SV2. However, unlike SV3, the FVs may also be ahead of SV2. So, the situation is precisely the same as in the case of two SVs in the flow, except that the sum limit must now be equal to $(N-3)$ since SV2 will always be followed by two other SVs, which also means that it cannot take two last positions in the TF. The sum of the vehicle speeds in such a flow can be written down using the number of vehicles following the first SV and their average speed determined from Eq. (3):

$$x \cdot (V_s + \Delta_s) + (V_s + \Delta_2) + (N - x - 1) \cdot \bar{V}_x.$$

In this situation, the number of vehicle orders when SV2 can be ahead of the other SVs in the flow is also can be considered through the use of expression $(N-2-x) \cdot (N-1-x)$. Then, by analogy with Eq. (9), the following equation for $\bar{V}_2$ can be written down:

$$\bar{V}_2 = \frac{\sum_{x=0}^{N-3} (N-2-x) \cdot (N-1-x) \cdot \frac{[x \cdot (V_s + \Delta_s) + (V_s + \Delta_2) + (N-x-1) \cdot \bar{V}_x]}{N}}{\sum_{x=0}^{N-3} (N-2-x) \cdot (N-1-x)}.$$

In multiplier $[x \cdot (V_s + \Delta_s) + (V_s + \Delta_2) + (N-x-1) \cdot \bar{V}_x]$, which is on the right of the numerator, the term in the middle, which contains only the constants V_s and Δ_2, can be taken out beyond the summation sign since the weighted average of the constant is equal to this constant itself. After that, the average speed $\bar{V}_x$ can be substituted with Eq. (3), that will result in the following:

$$\bar{V}_2=\frac{V_s+\Delta_2}{N}+\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{\left(x\cdot(V_s+\Delta_s)+(N-x-1)\cdot\left[V_s+\frac{N-2-x}{2\cdot(N-x-1)}\cdot\Delta_2\right]\right)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=\frac{V_s+\Delta_2}{N}+\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{\left(x\cdot\Delta_s+V_s\cdot(N-1)+\frac{N-2-x}{2}\cdot\Delta_2\right)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=\frac{V_s+\Delta_2}{N}+\frac{V_s\cdot(N-1)}{N}+\frac{1}{N}\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\left(x\cdot\Delta_s+\frac{N-2-x}{2}\cdot\Delta_2\right)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=V_s+\frac{\Delta_2}{N}+\frac{1}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\left(x\cdot\Delta_s+\frac{N-2-x}{2}\cdot\Delta_2\right)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=V_s+\frac{\Delta_2}{N}+\frac{\Delta_s}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}+\frac{\Delta_2}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{N-2-x}{2}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=\bar{V}_1+\frac{\Delta_2}{N}+\frac{\Delta_2}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{N-2-x}{2}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=\bar{V}_1+\frac{\Delta_2}{N}+\frac{\Delta_2}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{N-2}{2}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}-\frac{\Delta_2}{2\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)},$$

$$\bar{V}_2=\bar{V}_1+\frac{\Delta_2}{N}+\frac{\Delta_2\cdot(N-2)}{2\cdot N}-\frac{\Delta_2\cdot(N-3)}{8\cdot N}=\bar{V}_1+\frac{\Delta_2}{N}\cdot\left(1+\frac{N-2}{2}-\frac{1}{2}\cdot\frac{N-3}{4}\right),$$

$$\bar{V}_2=\bar{V}_1+\frac{3\cdot\Delta_2\cdot(N+1)}{8\cdot N}. \qquad (20)$$

The formula for the third term in Eq. (7) can be derived based on the analogy with the case of two SVs in a TF (Eq. (6)). In the situation under consideration, which is presented in Fig. 3, the case with two SVs arises for $(N-x-2)$ vehicles behind position x in the flow. The maximum speed for these $(N-x-2)$ vehicles should be taken equal to V_3. By analogy with Eq. (5), the sum of vehicle speeds in the flow will be as follows:

$$x \cdot (V_s + \Delta_s) + (V_s + \Delta_3) + (N - x - 1) \cdot \tilde{V}_x , \tag{21}$$

where $\tilde{V}_x$ is the average speed of vehicles following the SV3 provided that it takes a position in the flow ahead of the other two SVs. The formula for this average speed ($\tilde{V}_x$) can be obtained from Eq. (6) by replacing N with $(N - x - 1)$ and proper changing of subscripts at designations Δ:

$$\tilde{V}_x = V_s + \frac{\Delta_3}{N - x - 1} \cdot \frac{N - x - 3}{3} + \frac{\Delta_2}{2 \cdot (N - x - 1)} \cdot \left(1 + \frac{N - x - 3}{3}\right). \tag{22}$$

Unlike the case of only two SVs in a TF, to obtain the formula for $\bar{V}_3$ in Eq. (7), it is necessary to use the average speed of the vehicles behind SV3 represented by not the Eq. (3), but Eq. (21) divided by the number of vehicles in the flow since Eq. (21) takes into account the presence of two SVs with a speed lower than the speed of the leading SV. Given the same number of vehicle orders when SV3 can be ahead of the other SVs in the flow as in the two previous cases with other leading SV, it is possible to write down the equation for $\bar{V}_3$ based on the analogy with Eq. (9):

$$\bar{V}_3 = \frac{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x) \cdot \frac{[x \cdot (V_s + \Delta_s) + (V_s + \Delta_3) + (N - x - 1) \cdot \tilde{V}_x]}{N}}{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x)} .$$

In multiplier $[x \cdot (V_s + \Delta_s) + (V_s + \Delta_3) + (N - x - 1) \cdot \tilde{V}_x]$ on the right of the numerator, the term in the middle, which contains only the constants V_s and Δ_3, can be taken out beyond the summation sign since the weighted average of the constant is equal to this constant itself. After that, average speed $\tilde{V}_x$ can be substituted with Eq. (22), that will result in the following:

$$\bar{V}_3 = \frac{V_s + \Delta_3}{N} + \frac{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x) \cdot \frac{[x \cdot (V_s + \Delta_s) + (N - x - 1) \cdot Y]}{N}}{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x)} ,$$

where $Y = V_s + \frac{\Delta_3}{N - x - 1} \cdot \frac{N - x - 3}{3} + \frac{\Delta_2}{2 \cdot (N - x - 1)} \cdot \left(1 + \frac{N - x - 3}{3}\right)$.

Removing the inner brackets in expression $[x \cdot (V_s + \Delta_s) + (N - x - 1) \cdot Y]$ in the numerator allows for eliminating the repetition of the value of the speed of SV1 in the flow:

$$\bar{V}_3 = \frac{V_s + \Delta_3}{N} + \frac{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x) \cdot \frac{\left(x \cdot \Delta_s + [N - 1] \cdot V_s + \frac{\Delta_3 \cdot (N - x - 3)}{3} + \frac{\Delta_2}{2} + \frac{\Delta_2}{2} \cdot \frac{N - x - 3}{3}\right)}{N}}{\sum_{x=0}^{N-3} (N - 2 - x) \cdot (N - 1 - x)} .$$

After that, it is possible to take out the constants that do not depend on the summation index beyond the summation sign:

$$\bar{V}_3=\frac{V_s+\Delta_3}{N}+\frac{[N-1]\cdot V_s}{N}+\frac{\Delta_2}{2\cdot N}+\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot\frac{\left(x\cdot\Delta_s+\frac{\Delta_3\cdot(N-x-3)}{3}+\frac{\Delta_2}{2}\cdot\frac{N-x-3}{3}\right)}{N}}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}.$$

Now, it is possible to divide the large fraction in the numerator into several terms:

$$\bar{V}_3=V_s+\frac{\Delta_3}{N}+\frac{\Delta_2}{2\cdot N}+\frac{\Delta_s}{N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}+\frac{\Delta_3}{3\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-x-3)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}+$$
$$+\frac{\Delta_2}{6\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-x-3)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}.$$

The simplification of the first fraction among those showing the ratio of the sums is presented in Eq. (18), and the following fractions of this type can be factorised to extract index x:

$$\bar{V}_3=V_s+\frac{\Delta_3}{N}+\frac{\Delta_2}{2\cdot N}+\frac{\Delta_s}{N}\cdot\frac{N-3}{4}+$$
$$+\frac{\Delta_3}{3\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-3)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}-\frac{\Delta_3}{3\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}+$$
$$+\frac{\Delta_2}{6\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot(N-3)}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}-\frac{\Delta_2}{6\cdot N}\cdot\frac{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)\cdot x}{\sum_{x=0}^{N-3}(N-2-x)\cdot(N-1-x)}.$$

Based on Eq. (18) and the fact that certain sums contain an index-independent value $(N-3)$, it is possible to simplify these sums as follows:

$$\bar{V}_3=V_s+\frac{\Delta_3}{N}+\frac{\Delta_2}{2\cdot N}+\frac{\Delta_s}{N}\cdot\frac{N-3}{4}+\frac{\Delta_3\cdot(N-3)}{3N}-\frac{\Delta_3}{3\cdot N}\cdot\frac{N-3}{4}+\frac{\Delta_2\cdot(N-3)}{6\cdot N}-\frac{\Delta_2}{6\cdot N}\cdot\frac{N-3}{4}.$$

Now, it is reasonable to rearrange the terms in the following way:

$$\bar{V}_3=V_s+\frac{\Delta_s}{N}\cdot\frac{N-3}{4}+\frac{\Delta_2}{2\cdot N}+\frac{\Delta_2\cdot(N-3)}{6\cdot N}-\frac{\Delta_2}{6\cdot N}\cdot\frac{N-3}{4}+\frac{\Delta_3}{N}+\frac{\Delta_3\cdot(N-3)}{3\cdot N}-\frac{\Delta_3}{3\cdot N}\cdot\frac{N-3}{4}.$$

After that, proper terms can be substituted with $\bar{V}_1$ from Eq. (19), and the equation for $\bar{V}_3$ can be finalised as follows:

$$\bar{V}_3=\bar{V}_1+\frac{\Delta_2}{2\cdot N}\cdot\left(1+\frac{N-3}{3}-\frac{N-3}{12}\right)+\frac{\Delta_3}{N}\cdot\left(1+\frac{N-3}{3}-\frac{N-3}{12}\right)=\bar{V}_1+\frac{\Delta_2}{2\cdot N}\cdot\left(1+\frac{N-3}{4}\right)+\frac{\Delta_3}{N}\cdot\left(1+\frac{N-3}{4}\right).$$

$$\bar{V}_3 = \bar{V}_1 + \frac{\Delta_2}{2 \cdot N} \cdot \frac{N+1}{4} + \frac{\Delta_3}{N} \cdot \frac{N+1}{4}. \quad (23)$$

Thus, all the terms in Eq. (7) are deduced – these are Eqs. (19), (20), (23) representing the flow speed in situations when SV1, SV2, and SV3 take a position ahead of the other SVs. After the substitution of these equations into Eq. (7), the latter one takes the following form:

$$\bar{V} = \frac{1}{3} \cdot \bar{V}_1 + \frac{1}{3} \cdot \left(\bar{V}_1 + \frac{3 \cdot \Delta_2 \cdot (N+1)}{8 \cdot N} \right) + \frac{1}{3} \cdot \left(\bar{V}_1 + \frac{\Delta_2}{2 \cdot N} \cdot \frac{N+1}{4} + \frac{\Delta_3}{N} \cdot \frac{N+1}{4} \right).$$

This substitution allows for finalising the formula for the average speed of a dense TF with three SVs:

$$\bar{V} = V_s + \frac{\Delta_s}{N} \cdot \frac{N-3}{4} + \frac{\Delta_2}{N} \cdot \frac{N+1}{6} + \frac{\Delta_3}{N} \cdot \frac{N+1}{12}. \quad (24)$$

This formula establishes the dependence of the average speed of the vehicles at the exit from a one-lane road section with no overtaking opportunities from the total number of these vehicles in the flow (platoon) and the speed of the three SVs.

DISCUSSION

The initial analysis of the formula in Eq. (24) indicates that with the increase in N, there is an increase in the influence of the second term in Eq. (24), which represents the contribution of the slowest vehicle to the average flow speed and a decrease in the influence of subsequent terms which reflect the contribution of the other SVs, Fig. 4.

This means that additional SVs moving at speeds above V_s have an effect on the average speed, which is less than that of the slowest SV, and the closer the speed of additional SVs to V_{max}, the lower the effect. This effect can be characterised as follows: when $N \to \infty$, the average platoon speed will not exceed V_s by more than $(\Delta_s/4 + \Delta_2/6 + \Delta_3/12)$.

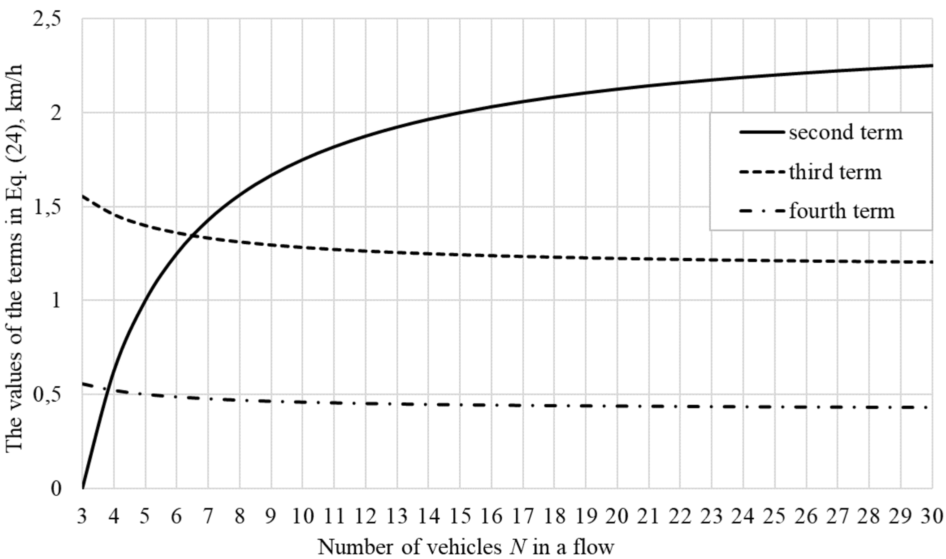

Figure 4 – Influence of the terms in the formula for the average speed of a dense TF with three SVs at different total number of vehicles in the flow (example for $\Delta_s = 10$ km/h, $\Delta_2 = 7$ km/h, $\Delta_3 = 5$ km/h)

It is also worth noting that when $N \to \infty$, the coefficients at terms $\Delta_s, \Delta_2, \Delta_3$ in Eq. (24) follow the reciprocals of the rectangular (pronic) numbers (1/4 = 1/2 · 1/2, 1/6, 1/12) [30, 31]. They can be interpreted as a characteristic of a gradual tangible decrease in the contribution of each subsequent SV to the average TF speed $\bar{V}$. This may mean that the SVs have a regular influence on the average flow speed since the infinite series of the reciprocals of the rectangular numbers sums to 1 ($\sum_{q=1}^{\infty}\frac{1}{q\cdot(q+1)}=1$ [30, 31]) just as the sum of the probabilities of random variable values.

CONCLUSIONS

One of the widely used tools to forecast urban TF parameters for engineering and scientific purposes is a FD of TF, which is based on statistical analysis of empirical data and their approximation with the most convenient and, whenever possible, simple functions. Many functions have been fitted to represent the FD but none clarifies the regularities reflected in the scattering of data points in the diagrams. This scattering also applies to the speed-density diagram, which indicates a decrease in speed with an increase in density. The results of studying the speed as a random variable also make a little contribution to clarification of these regularities and allow for concluding that the distribution of speed values plotted in the leftmost part of the speed-density diagram is known – this is a normal distribution which is beyond question in the scientific and engineering community – and the distribution of speeds plotted in the rightmost part of the diagram is characterised by ambiguity. To study the latter distribution, it is reasonable to carefully examine the process of forming the average vehicle speed value and consider practically possible situations of vehicle movement in a dense TF. Corresponding research has already been started in papers [28] and [29], which consider a dense platoon of vehicles, where all vehicles are willing to drive at the same maximum permitted speed except for one and two SVs that limit the speed of the others.

The current paper extends the mentioned research and presents the formula for calculating the average speed of a platoon, where three SVs limit the speed of other drivers who would drive faster in free-flowing conditions. Taken together, this provides a sound basis for generalising the obtained formulas to derive a formula for calculating the average speed of a TF with any number of SVs. Knowledge of this formula will be a starting point for understanding the change in the average value of the normally distributed free-flow speed caused by the increase in TF density and the corresponding complication of traffic conditions.

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***Горбачов П.Ф., Свічинський С.В.* Середня швидкість щільного транспортного потоку з трьома повільними автомобілями на ділянці дороги з обмеженими можливостями маневрування**

Прогнозування характеристик міських транспортних потоків залишається серед актуальних задач для науковців і практиків по всьому світу. Вирішити її допомагає фундаментальна діаграма транспортного потоку, яка являє собою статистично встановлений зв'язок між його параметрами, зокрема, між швидкістю і щільністю. Фундаментальна діаграма показує зниження швидкості потоку зі збільшенням його щільності, і серед пояснень цьому є вимушене підлаштування водіїв під швидкість повільніших автомобілів попереду. Це безумовно впливає на коливання швидкості і вказує на доцільність її розгляду як випадкової величини зі своїм середнім значенням та розподілом при різних умовах руху. Для вільного потоку загальновизнаним є нормальний розподіл швидкостей транспортних засобів, в той час як питання про те, який розподіл використовувати для опису швидкостей у щільному потоці, залишається відкритим. Поточне дослідження є внеском у прояснення даного питання і вибудоване навколо процесу формування середнього значення швидкості однотипних транспортних засобів у найбільш щільному транспортному потоці на ділянці дороги, де неможливі обгони і випередження. Пошук цього середнього значення був розпочатий з розгляду найпростіших ситуацій руху пачки автомобілів, де всі автомобілі бажають їхати з однаковою найбільшою незабороненою швидкістю, окрім одного та двох ПА, які обмежують швидкість інших. Дана стаття є їх логічним продовженням і присвячена отриманню формули для розрахунку середньої швидкості транспортного потоку для випадку трьох повільних автомобілів у ньому. Все разом це створює ґрунтовну основу для узагальнення отриманих залежностей з метою виведення формули для розрахунку середньої швидкості потоку з будь-якою кількістю повільних автомобілів. Знання такої залежності послужить відправною точкою для розуміння спричиненої щільним потоком зміни як середнього значення, так і розподілу швидкості руху, котра у вільних умовах описується нормальним законом.

Ключові слова: швидкість руху, середня швидкість, транспортний потік, щільність потоку, смуга руху, умови руху, аналітичне моделювання, транспортна система.

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DOI 10.36910/automash.v2i25.1905