УДК 656.2:519.876
UDK 656.2:519.876

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STUDY OF THE DYNAMICS OF EXPORT METAL PRODUCTS FREIGHT FLOWS AND THEIR MATHEMATICAL MODELING UNDER CONDITIONS OF AN UNSTABLE ENVIRONMENT

The article examines how export transportation of metal products by rail changed under conditions of abrupt transformations in the external environment. Particular attention is paid to the state of the transportation process before the onset of large-scale disruptive events and to the way it evolved under their influence, reflecting its gradual adaptation to new operating conditions.

For quantitative assessment, the following statistical indicators were selected: mean value, standard deviation, and coefficient of variation. This approach made it possible not only to determine the transportation volume levels but also to identify the internal irregularity of transportation over time, particularly the periods when fluctuations became sharper and less predictable.

The results indicate that the transportation process underwent significant changes during the study period. Initially, it functioned relatively steadily; however, it later shifted into a phase of pronounced instability. Subsequently, signs of a new operating regime gradually emerged, better adapted to existing conditions. At the same time, the variability of indicators increased, and the nature of their fluctuations changed. Changes in the intra-annual distribution of transportation volumes were also observed, further confirming the non-stationary nature of the process.

To describe these changes, a mathematical model combining a trend component and fluctuations with variable parameters is proposed. This approach makes it possible to simultaneously account for the general development trend and short-term deviations. Model validation demonstrated that it reproduces real dynamics with sufficient accuracy (mean relative error 11,81%, $R^2 = 0{,}55$), indicating its applicability as a practical analytical tool.

Overall, the obtained results contribute to a better understanding of transportation behavior under unstable conditions and may be useful for forecasting and decision-making in logistics management, particularly in situations characterized by uncertainty and variability.

Keywords: freight flows, export transportation, railway transport, mathematical modeling, seasonality, non-stationary process, transport system, variation, forecasting.

INTRODUCTION

In modern conditions, transport systems increasingly deviate from the traditional perception of stable and predictable structures. They are becoming more dependent on external factors, including economic and political conditions, as well as infrastructure status, and respond to them quite sensitively. What only a few years ago appeared to be a relatively organized process has now become subject to frequent changes. Under such circumstances, the issue of transportation stability becomes particularly important, as transport serves as the link between production, supply, and consumption.

Export transportation of metal products is one of the sectors where these changes are felt most acutely. On the one hand, it is an important component of the economy, generating financial inflows and supporting the functioning of entire industries. On the other hand, this segment is among the first to react to any disruptions. Changes in transportation routes, restrictions on specific sections, or infrastructure failures are immediately reflected in transportation volumes and operational rhythms. As a result, the system becomes vulnerable even to relatively local disturbances.

Under martial law conditions, this vulnerability is further intensified. Transport operations are accompanied by constant uncertainty, which is clearly reflected in transportation dynamics. Volumes may change sharply, sometimes without any obvious reason at first glance. Irregularity increases, with periods of relative calm alternating with sudden rises or declines. Seasonal patterns previously considered typical no longer function in the same way or manifest differently. At the same time, new logistics solutions emerge - often forced and sometimes temporary - but it is precisely these measures that enable the system to adapt, at least partially, to the new conditions.

LITERATURE REVIEW AND PROBLEM STATEMENT

In contemporary transport system research, the focus is increasingly shifting from transportation volumes themselves to the way these volumes behave over time. This trend is quite natural, as freight flows prove to be far less stable than assumed by classical approaches. Even over short time intervals, their dynamics may change rather sharply, and such changes cannot always be adequately explained by standard models [1, 2].

Traditional time series analysis methods, such as ARIMA and exponential smoothing, remain fundamental analytical tools. However, their capabilities are limited by the assumption of relative process stability. When the data structure changes, the accuracy of such models decreases significantly. For this reason, they are increasingly being combined with machine learning methods [3, 4, 5]. This makes it possible to better capture complex dependencies that are not always suitable for linear description and improves model flexibility in practical applications [6, 7, 8].

This tendency is particularly evident in railway transportation. In this field, hybrid approaches have effectively become standard analytical tools. Most commonly, these are combinations of ARIMA and neural networks, where one part of the model is responsible for capturing the general trend, while the other accounts for more chaotic short-term fluctuations [3, 8]. It is precisely these fluctuations that usually create the greatest forecasting difficulties. Nevertheless, such models demonstrate better adaptability to data changes compared with classical approaches [6].

Seasonality should also be considered separately. In theoretical models, it is usually represented as stable and repetitive; however, real transport processes are more complex. Seasonal effects may vary both in intensity and in form. In some cases, they become almost imperceptible, while in others they intensify sharply. These features are better captured by modern exponential smoothing approaches with extended seasonal structures [4], as well as by grouped time series forecasting models [5].

The problem becomes even more complicated when the system is influenced by external factors. Crisis events, disruptions in logistics chains, and changes in transportation conditions all directly affect data behavior. In such cases, structural breaks and shifts in operating regimes are often observed and can be analyzed using change-point detection methods [9]. Similar effects are also actively studied in the context of supply chain resilience and adaptation [10, 11, 12].

In practice, this is reflected in the fact that models previously demonstrating acceptable accuracy begin to generate significant errors. The issue is not limited to changes in the trend alone - the structure of variation itself changes, and the assumption of stationarity is no longer valid [2, 5].

Machine learning methods partially address this issue, as they are better suited to capturing complex nonlinear relationships. However, they present another limitation: the results of such models are considerably more difficult to interpret [7]. Therefore, increasing attention is being paid to forecast explainability approaches, particularly the SHAP method, which makes it possible to evaluate the contribution of each variable to the model output [13].

When transport systems are considered more broadly, it becomes evident that they operate in a continuously changing environment and respond quite sensitively to external influences. This is especially apparent in the analysis of large-scale transport datasets, where even minor changes in operating conditions may cause noticeable fluctuations [14]. This phenomenon is most pronounced in export transportation of metal products, where freight flow dynamics are particularly sensitive to external factors.

As a result, a typical problem for this field emerges: some models describe long-term behavior effectively, while others better capture short-term fluctuations. Combining these aspects within a single approach remains challenging, yet this is precisely the direction in which current research is developing [3, 5].

The reviewed literature also confirms that classical time series models are based on the assumption of process stability. However, under real-world conditions, transport systems undergo continuous structural changes, which require more adaptive modeling approaches [4, 9].

Within the framework of this study, attention is focused not only on forecasting but also on analyzing how the structure of fluctuations changes over time. This makes it possible to gain a deeper understanding of freight flow behavior and to describe it more accurately under unstable conditions.

PURPOSE AND OBJECTIVES OF THE STUDY

The aim of the study is to identify patterns in changes in export transportation of metal products and to develop a model capable of adequately describing their dynamics under unstable conditions.

To achieve this aim, the following objectives were defined: to analyze transportation volumes for the period 2021 - 2025; to assess the stability level of the studied process; to identify the nature of key changes in transportation dynamics; to investigate the transformation of the seasonal structure; to develop a mathematical model of freight flows; to verify the model based on actual data; and to evaluate forecasting accuracy as well as the potential for further model improvement.

RESEARCH RESULTS

The object of this study is the process of export transportation of domestically produced metal products by railway transport. To assess the stability of this process, statistical indicators were used, including the mean value, standard deviation, and coefficient of variation of transportation volumes (Table 1).

The application of these indicators makes it possible not only to characterize the overall level of transportation volumes but also to assess the intensity of their fluctuations over time. The analysis was conducted for two characteristic periods: the pre-war period (2021) and the period of operation under martial law conditions (2022 - 2025). This approach is consistent with methods used in the analysis of transport systems under crisis conditions and makes it possible to trace the transformation of the transportation process under the influence of external destabilizing factors.

These factors primarily include damage to transport and energy infrastructure caused by hostile attacks, changes in logistics routes, and restrictions on the use of rolling stock.

Table 1 – Statistical indicators of transportation process stability for 2021 - 2025

Year	Mean	Standard deviation	Coefficient of variation	Notes
2021	1 310	95,22	7,27	stable process
2022	456,6	376,26	82,41	severe instability
2023	378,8	44,59	11,77	moderate fluctuations
2024	430,7	62,16	14,43	moderate fluctuations
2025	486,8	76,71	15,76	moderate fluctuations

The analysis of the indicators presented in Table 1 demonstrates a significant change in the nature of the transportation process during the study period. In 2021, transportation was characterized by a high level of stability, as confirmed by the low coefficient of variation (7,27%), which is typical of relatively predictable transport system operation.

In 2022, a sharp increase in the variability of transportation volumes was observed, with the coefficient of variation reaching 82.41%. Such dynamics indicate substantial disruptions in transport system functioning caused by wartime factors, including attacks on energy infrastructure, changes in transportation routes, and restrictions on export opportunities.

Beginning in 2023, a tendency toward partial stabilization of the transportation process can be observed. The coefficient of variation decreased to 11,77% and remained within the range of 14 - 16% during 2024 - 2025, corresponding to a moderate level of fluctuations. This trend indicates not a return to pre-war conditions, but rather the formation of an adaptive operating mode of the transport system.

Figure 1 presents the dynamics of transportation volumes, illustrating the transition from a relatively uniform distribution in the pre-war period to a sharp decline and increased irregularity in 2022, followed by gradual recovery in subsequent years.

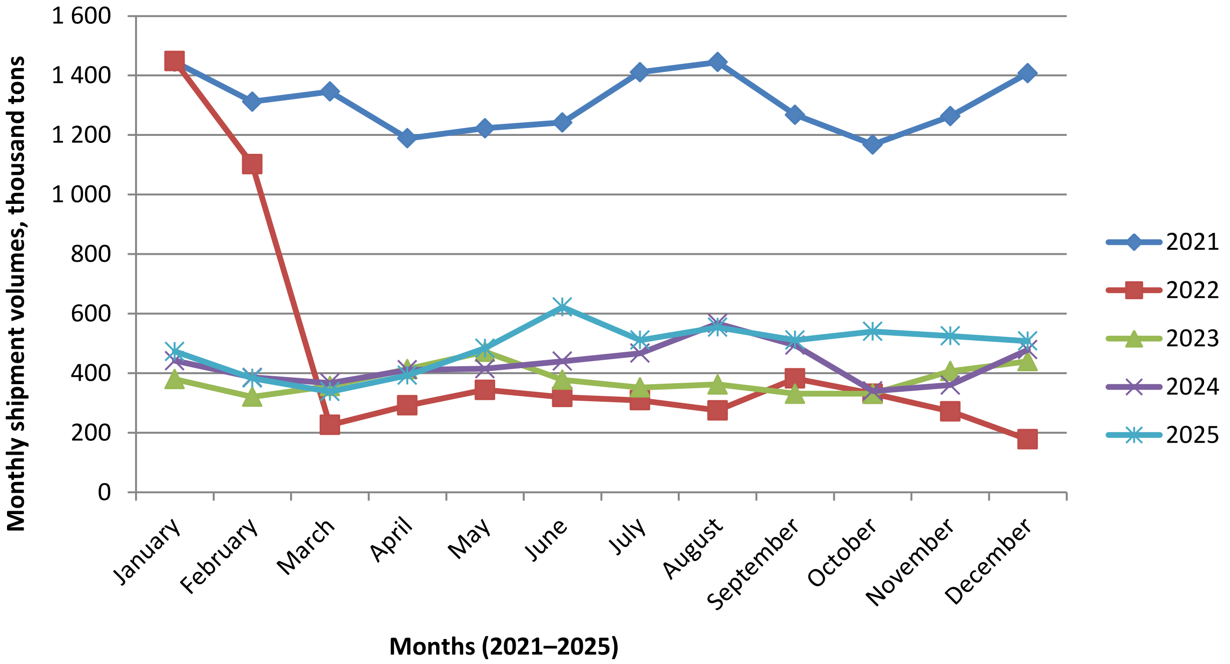

Figure 1 - Dynamics of transportation volumes by month (2021 - 2025)

The analysis of the graphical relationship confirms the identified trends: following a sharp decline in transportation volumes during the crisis period, their gradual recovery can be observed throughout 2023–2025.

The obtained results are consistent with the analysis of freight flow seasonality indices (Figure 2), which reflect the transformation of the intra-annual transportation structure. In 2022, the seasonal pattern experienced significant disruption, while in subsequent years a new seasonality model emerged, adapted to the changed operating conditions of the transport system.

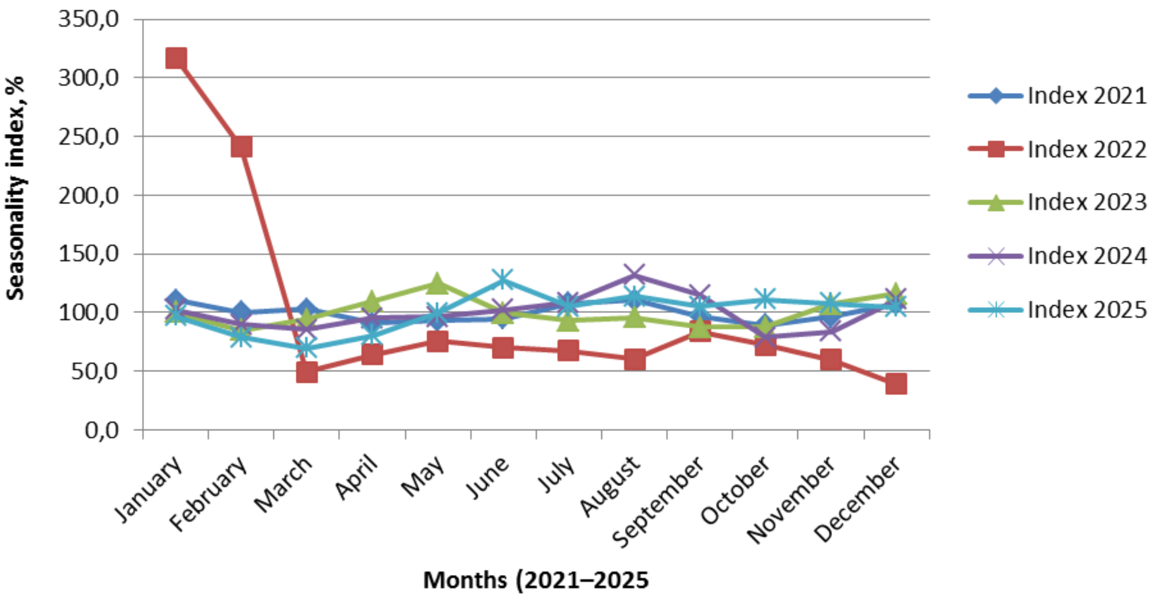

Figure 2 - Comparison of freight flow seasonality indices

Thus, the study results make it possible to distinguish three characteristic stages in the development of the transportation process: a stable pre-war period, a phase of sharp destabilization, and a stage of adaptation to new operating conditions.

Changes in the intra-annual transportation structure indicate the non-stationary nature of the process, which combines a long-term development trend with seasonal fluctuations of varying intensity.

These features determine the need for further development of a mathematical model capable of adequately reflecting the identified freight flow dynamics and accounting for the influence of external factors.

Mathematical model of freight flows.

To describe changes in freight flows over time, a mathematical model is considered that simultaneously accounts for the general development trend of transportation volumes and their characteristic seasonal fluctuations [11].

Let x denote the period number (month), and $f(x)$ - the freight flow volume at the corresponding time. In this case, the relationship can be expressed as:

$$f(x) = 90 \cdot e^{-0{,}028x} \cdot \sin(0{,}75x + 0{,}006x^2) + 4{,}8x + 312 \qquad (1)$$

Parameter identification of the mathematical model was carried out using the least squares method, which was chosen due to the need to approximate the empirical time series with an analytical function and to minimize deviations between simulated and actual transportation values. This method is appropriate under conditions of a limited sample size (42 monthly observations), ensures the stability of parameter estimates, and preserves their interpretability.

It should be noted that the sample size limits the possibility of applying more complex stochastic or neural network-based models; however, it is sufficient for constructing an interpretable analytical dependence focused on analyzing the nature of dynamics rather than high-precision forecasting.

The application of the least squares method is consistent with the generally accepted practice of analyzing non-stationary transport processes and is a justified tool for estimating parameters of combined regression models [3, 13].

Unlike simplified approaches, the proposed model combines several components, each of which has its own substantive interpretation.

The linear component reflects the gradual change in transportation volumes over the long term. Its presence may be associated with the development of transport infrastructure, changes in demand structure, or the overall dynamics of economic activity.

At the same time, the oscillatory component captures seasonal effects, which in practice are rarely strictly periodic. In the model, these effects vary over time: the amplitude gradually decreases, which may indicate a certain stabilization of the process or adaptation of the system to new operating conditions.

Attention should also be paid to the argument of the sinusoidal function—it is nonlinear. This allows for changes in oscillation frequency to be taken into account, which is typical for real transport processes, especially under unstable external conditions.

Thus, the proposed dependence does not merely approximate the data but reflects the key features of freight flow dynamics.

To verify the appropriateness of the model, it was further tested against actual data.

Modeling Results and Validation.

After constructing the mathematical model, its validation was carried out by comparing the calculated values with the actual data. This approach is standard in time series model evaluation problems [12, 14].

Figure 3 presents the dynamics of actual and simulated transportation volumes. Overall, it can be noted that the model reproduces the main regularities of the process quite well: it captures both the general trend in the indicator and its seasonal fluctuations. At the same time, in certain periods, deviations between observed and modeled values can be seen, which is natural, as the model does not account for all possible external influences.

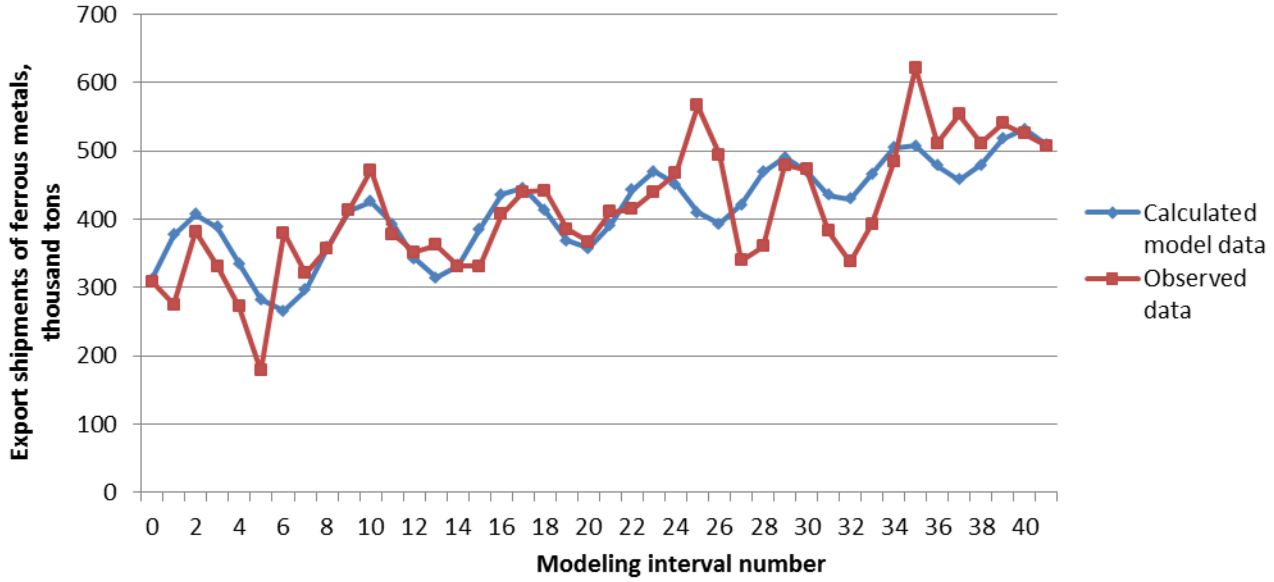

Figure 3 - Dynamics of actual and modeled freight flow values

For a quantitative assessment of approximation accuracy, a set of aggregate indicators was used. The coefficient of determination is $R^2 = 0{,}55$, which corresponds to a moderate level of agreement between the model and the observed data. This value is explained by the high variability of the studied process and the presence of external factors that are not formalized within the model, which is typical for non-stationary transport systems [7]. The mean absolute error equals 45,41, while the root mean square error is 59,84. The mean relative error is 11,81%, which can be considered an acceptable result for problems of this type.

Additionally, the behavior of model residuals was analyzed (Figure 4). In most cases, they fluctuate around the zero line, indicating the absence of systematic bias. At the same time, several significant deviations are observed, which are random in nature and are likely associated with the influence of external factors not accounted for in the model.

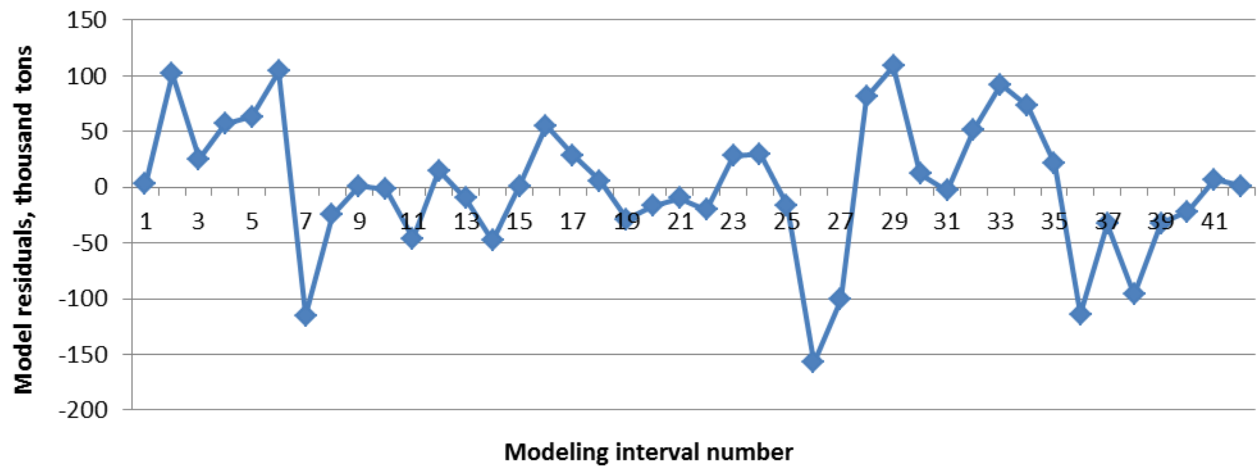

Figure 4 - Residual analysis of the model

The distribution of residuals (Figure 5) does not reveal any critical anomalies: the shape of the histogram is generally symmetric, although a full correspondence to a normal distribution is not observed.

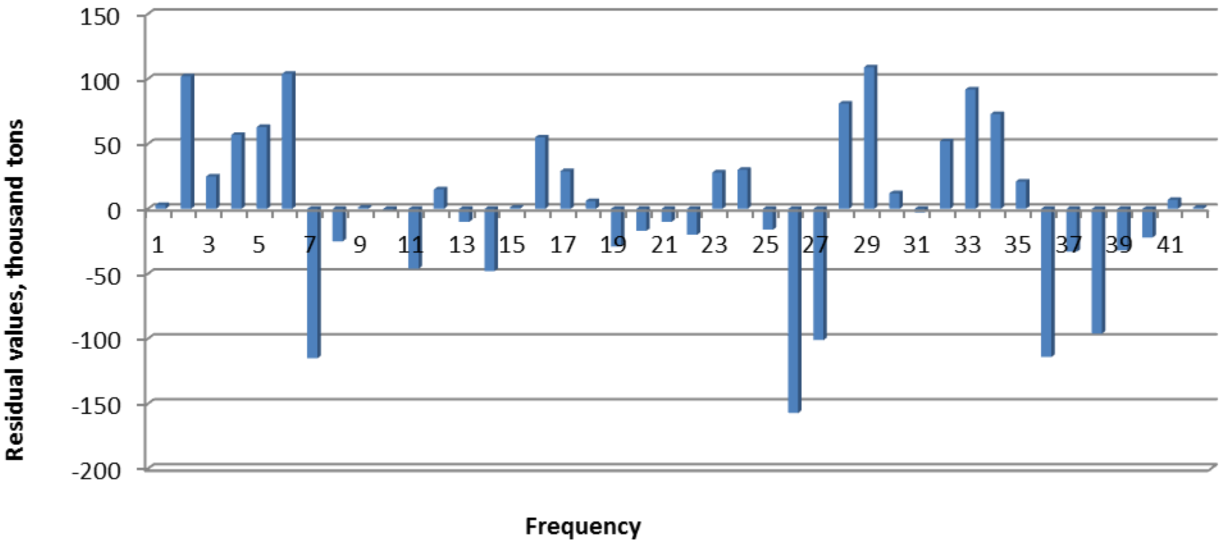

Figure 5 – Distribution of model residuals

Summarizing the results, it can be concluded that the developed model is suitable for the analysis and approximate forecasting of freight flows. At the same time, to improve accuracy, it is advisable to extend the model by incorporating additional factors, primarily those responsible for sharp fluctuations in transportation volumes.

DISCUSSION OF RESEARCH RESULTS

The obtained results make it possible to generalize the observed changes in the transportation process. The sharp increase in variability in 2022 effectively indicates a loss of system stability and a disruption of its usual operating regimes. The subsequent reduction in fluctuations should not be interpreted as a return to the previous state; rather, it reflects a transition to a new, adapted operating regime.

A noticeable change is also observed in the seasonal structure. This indicates that the process has become non-stationary and that its internal organization has changed.

The proposed model only partially captures these changes, as it does not account for external factors influencing transportation dynamics. In practical terms, it reproduces the general behavior of the time series reasonably well and can therefore be used for approximate forecasting; however, it requires further refinement when higher accuracy is needed.

As the analysis shows, transportation dynamics are not homogeneous: they combine slow long-term changes with pronounced short-term fluctuations. Statistical evaluation made it possible to clearly identify the turning point in 2022, when stability was effectively lost, followed by a transition to a different system operating mode.

Overall, the model reflects the main characteristics of the process - trend, seasonality, and time-varying amplitude of fluctuations-thereby enabling a shift from a descriptive representation of the phenomenon to a more formalized analytical framework.

CONCLUSIONS

The conducted study has shown that transportation dynamics are complex and heterogeneous. They combine long-term changes with short-term fluctuations, which may differ significantly in intensity.

Three main stages of process development have been identified: stable, crisis, and adaptive. The most pronounced changes occurred in 2022, after which the system gradually transitioned to a new operating regime.

The proposed model allows for the description of the main features of the process, including trend, seasonality, and time-varying amplitude of fluctuations. At the same time, it requires further improvement, particularly through the inclusion of external factors. In future research, it would be appropriate to consider incorporating exogenous variables such as infrastructure, energy, and regulatory factors.

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Турпак С.М., Муковська Д.Я. **Дослідження динаміки експортних вантажопотоків металопродукції та їх математичне моделювання в умовах нестабільного середовища**

У статті простежено, як змінювалися експортні перевезення металопродукції залізницею в умовах різких змін зовнішнього середовища. Акцент зроблено на тому, яким був стан транспортного процесу до початку повномасштабних подій і як саме він почав змінюватися під їх впливом, тобто як відбувалася його поступова перебудова.

Для кількісної оцінки обрано такі статистичні індикатори - середнє значення, стандартне відхилення та коефіцієнт варіації. Такий підхід дав змогу не лише зафіксувати рівень перевезень, а й побачити їхню внутрішню «нерівність» у часі, зокрема виявити періоди, коли коливання ставали більш різкими та менш передбачуваними.

Результати свідчать, що транспортний процес за досліджуваний період зазнав відчутних змін: якщо спочатку він функціонував відносно рівномірно, то згодом перейшов у фазу різкої нестабільності. Після цього поступово почали проявлятися ознаки нового режиму роботи, більш пристосованого до наявних умов. При цьому зросла мінливість показників, а характер їх коливань став іншим. Також звертає на себе увагу зміна внутрішньорічного розподілу перевезень, що додатково підтверджує нестаціонарність процесу.

Для опису цих змін запропоновано математичну модель, у якій поєднано трендову складову та коливання зі змінними параметрами. Це дозволяє одночасно враховувати загальну тенденцію розвитку і короткострокові відхилення. Перевірка показала, що модель досить добре відтворює

реальну динаміку (середня відносна похибка становить 11,81 %, $R^2 = 0{,}55$), тобто може використовуватися як робочий інструмент аналізу.

У цілому отримані результати дають можливість краще зрозуміти поведінку перевезень у нестабільних умовах і можуть бути корисними при прогнозуванні та прийнятті управлінських рішень у сфері логістики, особливо тоді, коли середовище залишається невизначеним і змінним.

Ключові слова: вантажопотоки, експортні перевезення, залізничний транспорт, математичне моделювання, сезонність, нестаціонарний процес, транспортна система, варіація, прогнозування.

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Дата надходження статті до видання: 01.04.2026
Дата прийняття статті до друку після рецензування: 30.04.2026
https://doi.org/10.36910/ha7dw842