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ANALYSIS OF METHODS FOR CALCULATING FRACTAL DIMENSION AS A TOOL FOR ENHANCING PERFORMANCE AND SAFETY CHARACTERISTICS OF AUTOMOTIVE TRANSPORT OBJECTS

This paper is devoted to developing a concept for the effective utilization of methods for analyzing and calculating the fractal dimension of models in two-, three-, and multidimensional spaces aimed at solving practical problems related to enhancing qualitative characteristics of automotive transport objects, transport infrastructure, and logistics. The most universal and computationally efficient method for calculating fractal parameters has been identified, opening new possibilities for optimizing modeling processes of real systems and significantly improving the quality of their analysis and forecasting.

Contemporary applied tasks, including those in transportation technologies, are actively progressing towards investigating processes within multidimensional spaces related to the dynamics of chaotic systems behavior. Searching for universal methods and algorithms to calculate fractal parameters for multiparametric object models within arbitrary-dimensional spaces, particularly those changing their properties over time, requires additional research for generalization.

The analysis of literature sources revealed that previous research still contains several unresolved issues regarding fractal analysis. These formed the basis for formulating the tasks addressed in this paper, including classification of methods for calculating fractal dimension as tools for solving practical problems across various scientific and technological fields; investigation of computational algorithms for determining fractal parameters of objects concerning their potential for generalization and use in spaces of arbitrary dimensionality; and identification of interdependencies and interrelations between fractal parameters of quasi-fractal and multifractal computational models and the technological, technical, and economic characteristics of objects and processes within the automotive industry. Based on these, ways to develop a practical method for calculating and analyzing fractal objects, characterized primarily by universality and standardization, have been outlined.

The research carried out in this study regarding the capabilities of the analyzed methods for calculating fractal dimensions, their classification according to defining characteristics, and the analysis of their algorithmic foundations led to the conclusion that the Box-Counting Method is the most effective and universal. It enables clear visualization of geometric complexity in models and their self-organization. The algorithm of this method is relatively simple and allows quantitative assessment of the hierarchical order and structural chaos level in studied objects, directly influencing accurate determination of relationships between fractal parameters and technological properties of real physical phenomena and processes.

A concept for improving the most universal Box-Counting method is proposed, with outlined directions for extending and deepening its theoretical, methodological, and algorithmic components. The idea of calculating fractal parameters for object models in spaces of arbitrary dimensions through continuous dimensionality reduction is introduced. Practically, this procedure resembles gradual stratification of a multidimensional object into separate layers of varying dimensionality, determining fractal parameters of these layers, and consolidating geometric data into a unified information framework. Such fractal scanning allows for adjusting individual layers, elements, or nodes within the model to further improve its technological and technical properties.

Keywords: fractal dimension, technological and performance characteristics of transport objects, multidimensional space, concept of interrelation between geometric parameters and qualitative properties of transport infrastructure objects.

INTRODUCTION

Modern scientific research in geometric analysis and modeling of complex systems increasingly utilizes the concept of fractal dimension. This is due to the inadequacy of traditional Euclidean models for accurately describing structures characterized by irregular, self-organized, or chaotic patterns. Fractal characteristics provide a more precise determination of geometric properties for such objects in two- and three-dimensional spaces, opening broad prospects for optimizing their technological qualities in various fields of science and engineering [1]. Particularly, in engineering, physics, biology, computer modeling, and materials science, fractal dimension is employed for analyzing complex surfaces, porous structures, turbulent flows, transport network quality, and even economic processes.

A significant challenge in solving this diverse range of practical problems is reducing the numerous research methods and tools for studying fractal properties of specific objects or phenomena to as few as possible. Therefore, a critical analysis of the most popular methods for calculating fractal dimensions, identifying their universal characteristics, classifying them, and seeking standardized algorithmic and software approaches for effective practical application constitute the primary relevant task. Moreover, it is crucial to consider that, for practical applications, correctly identifying the interrelation between fractal parameters and technological properties of the objects studied is even more important than the accuracy of calculating fractal characteristics themselves.

Additionally, contemporary applied tasks, including those in the field of transportation technologies, increasingly focus on investigating processes in multidimensional spaces related to the dynamics of chaotic systems. Finding universal methods and algorithms for calculating fractal parameters of multiparametric object models within spaces of arbitrary dimensionality, particularly those whose properties may vary over time, requires further research for generalization. Consequently, analyzing existing methods, improving them, and adapting them for fractal calculation of multidimensional objects represents the second relevant task, holding significant potential for applications in automotive transport and logistics.

LITERATURE REVIEW AND PROBLEM STATEMENT

The search and analysis of literature sources regarding the aforementioned issues were conducted in the following directions: contemporary methods for calculating the fractal dimension of objects in two-dimensional and higher-dimensional spaces; approaches for evaluating geometric properties of internal, non-visible areas of spatial fractal structures; and interrelations and mutual influences of fractal parameters of investigated objects on the enhancement of their technological characteristics.

The study [2] investigates the efficiency of the perimeter-area method, introduced by Mandelbrot, for determining the fractal dimension of complex geometric objects such as aggregate clusters. The publication emphasizes the importance of correctly choosing internal and characteristic measurements when applying this method, particularly for cluster-type objects. However, while highlighting caution in applying the perimeter-area method to various object types, the work does not provide detailed recommendations for its use with other complex structures.

Multifractal objects are the subject of study in [3], where the authors explore two mass-oriented methods for calculating generalized fractal dimension, particularly for a one-dimensional generalized Cantor set. They demonstrate these methods provide more accurate results compared to the box-counting method. Although mass-oriented methods exhibited superior results in a specific case, the authors acknowledge the necessity for further research to determine their limitations and applicability to other types of fractal structures.

Work [4] presents a fractal dimension calculation method based on the box-counting approach, incorporating mathematical definitions for cell dimensions and intervals. The authors generate fractal objects and define cells according to mathematical principles, reducing errors associated with traditional approaches. Despite improved accuracy, further research is necessary to evaluate its effectiveness for complex natural objects and under noisy conditions.

Publication [5] investigates the use of correlation dimension estimation in simulated phase spaces to identify characteristic relations in various dynamic systems. The method is presented as a rapid and reliable tool for detecting causal interactions in systems dominated by deterministic dynamic processes. However, the method requires further validation with real data and under conditions where stochastic processes significantly influence system dynamics.

Study [6] focuses on examining the geometry of rain- and cloud-formed areas identified using satellite and radar data. The author employs area-perimeter relationships to analyze the fractal form of these regions, enhancing understanding of their properties and behavior. Although the research allows extracting geometric information of cloud and rain areas, it does not correlate these findings with parameters influencing climate models and forecasts.

The authors in [7] propose a three-dimensional box-counting method for analyzing fractal characteristics of modern urban structures. They applied this method to assess the two-dimensional and three-dimensional fractal properties of urban matrices, aiding in understanding the complexity and hierarchy of urban environments. The research investigated the impact of fractal dimension on urban infrastructure development, including transportation networks, and its potential use in sustainable urban planning. However, the effect of varying scales and data resolutions on analysis results was not considered, an aspect essential for practical applications in urban studies.

In [8], the authors developed a three-dimensional road surface model using the box-counting method to determine fractal dimensions of random height points on road pavements. This provided new parameters for assessing pavement conditions and their influence on vehicle dynamics. However, results did not integrate fractal parameters into practical applications nor linked them with existing road quality evaluation standards. The impact of different pavement types on analysis results was also not specified.

Publication [9] evaluates the efficiency of minimal coverage in calculating fractal dimensions for river network schemes on maps. The authors proposed a methodology allowing more precise fractal dimension determinations, considering object complexity. However, detailed recommendations for automating minimal coverage, crucial for processing large datasets of river network imagery, were lacking. Additionally, generalizing results for spatial geometric structures of similar types proved challenging.

The literature analysis indicated several gaps that form the basis for defining the objectives of the current research. These include a lack of comprehensive approaches in the analyzed studies for using methods to calculate fractal properties of objects as numerical indicators of their qualitative and safety attributes; absence of research on calculating fractal characteristics independent of the modeling spaces; and undeveloped conceptual principles regarding interrelations and impacts of fractal properties on the technical quality of the analyzed processes.

RESEARCH AIM AND OBJECTIVES

The aim of this research is to develop a concept for effectively utilizing methods of analyzing and calculating the fractal dimension of models in two-, three-, and multidimensional spaces to solve practical tasks related to enhancing the qualitative characteristics of automotive transport objects, transport infrastructure, and logistics. Identifying the most universal and computationally efficient methods for calculating fractal parameters will open new opportunities for optimizing modeling processes of real systems and improving the quality of their analysis and forecasting.

To achieve this aim, the following objectives must be addressed:

1. Classify methods for calculating fractal dimension as tools for solving practical tasks in various fields of science and technology.

2. Investigate computational algorithms for determining fractal parameters of objects, focusing on their potential for generalization and application in spaces of arbitrary dimensionality.

3. Define the interrelations and mutual influences between fractal parameters of quasi-fractal and multifractal computer models and the technological, technical, and economic characteristics of automotive industry objects and processes. On this basis, outline the pathways for developing a practical method for calculations and analyses of fractal objects, characterized by universality and standardization in spaces of arbitrary dimensions.

The object of this research comprises methods for determining the fractal dimension of objects in spaces of arbitrary dimensionality.

The subject of the research is the specifics of applying fractal methods to analyze complex geometric models and their influence on assessing technological characteristics of studied objects (particularly automotive industry objects).

RESEARCH RESULTS

1. Classification of Methods for Calculating Fractal Dimension as Tools for Solving Practical Problems

Based on the analysis of contemporary approaches to calculating fractal dimensions and the defined research objectives, the methods are classified according to the following criteria: 1) complexity of method implementation and required computational resources; 2) dimensionality of the space in which fractal parameters of the object can be calculated using a specific method; 3) universality of the method and standardization of its calculation algorithms.

Classification based on the **complexity of method implementation and computational resources required** for practical use helps ensure optimal method selection depending on available computational capabilities, the type of input data, and the required accuracy.

Simple methods (Fig. 1) (low computational complexity). These methods are easy to implement and require minimal computational resources. They are suitable for processing small datasets or quick (approximate) estimates of fractal dimensions. Such methods primarily include:

– Box-Counting Method. The most widespread method, based on counting occupied and unoccupied cells after covering the object with grids of different scales. Easy to implement, this method is commonly used for analyzing the fractality of planar contours, textures, and porous structures, primarily in two-dimensional spaces.

– Perimeter-Area Method. Used for 2D objects, fractal dimension is evaluated through the perimeter-to-area ratio. Primarily applied to linear geometric figures, coastlines approximated by polygons, and various natural biological structures.

– Mass-Radius Method. This method assesses self-similarity in simple objects by analyzing the mass (number of zero-dimensional points) within circles of a certain radius. It is relatively easy to implement but requires effective selection of circle centers and result-averaging algorithms.

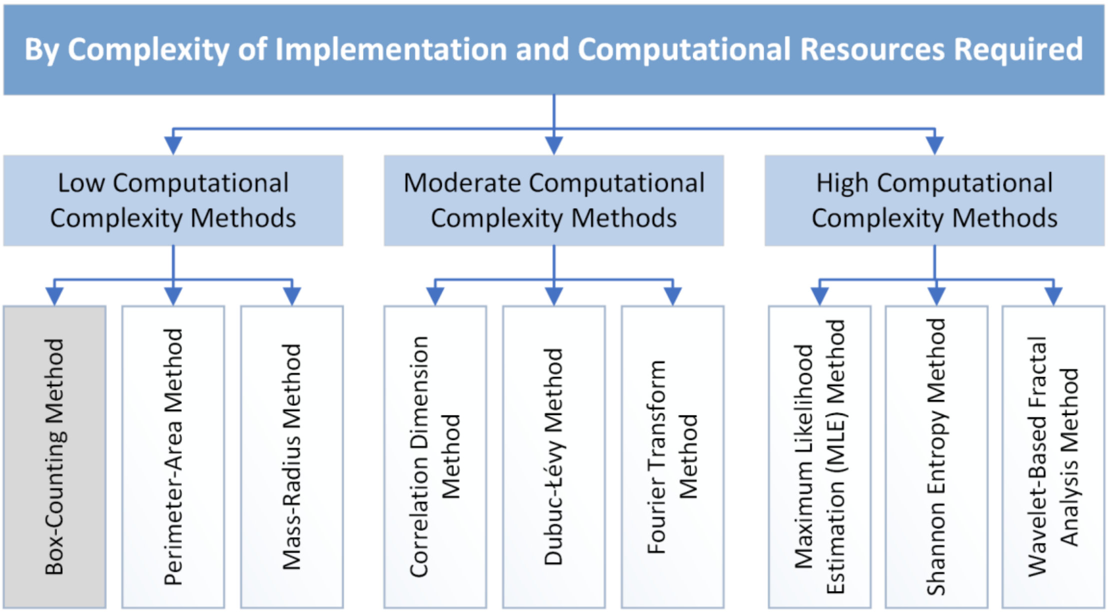


Fig. 1. Classification of fractal calculation methods by algorithm complexity

Moderately complex methods (Fig. 1). These methods require more complex algorithms, various statistical computations, or specific approaches to selecting parameters of studied objects:

– Correlation Dimension Method. Used to analyze interactions between fractal points through statistical correlations. The method involves calculating distances between large sets of points, complicating computations in large-scale systems. It can be used for chaotic system analysis, fluid dynamics, and clustering data in fractal models.

– Dubuc-Łévy Method. Determines local dimensions of objects by calculating scale variations for different regions. It is specialized and can be used for fractals with heterogeneous structures.

– Fourier Transform Method. Based on spectral analysis to estimate fractal properties. Requires continuous frequency data transformation, increasing complexity. Selectively applicable in physics, image processing, and fractal turbulent flow analysis.

Complex methods (Fig. 1) (high computational complexity). These methods are the most accurate for determining fractal dimensions but demand significant computational resources, specialized software, or machine learning:

– Maximum Likelihood Estimation (MLE Method). This method applies statistical approaches to estimate fractal structure distribution parameters. Suitable for analyzing biological systems or financial markets where calculation accuracy is crucial. However, it involves repeated probability distribution computations and complex optimization algorithms.

– Shannon Entropy Method. Uses entropy concepts to evaluate fractal dimensions, requiring calculations of multiple multidimensional probability distributions. Typically used only for neural networks analysis or signals with complex structures.

– Wavelet-Based Fractal Analysis Method. Employs wavelet transforms to analyze scale dependencies in fractal structures. Applied in medical research, turbulence analysis, and neuroscience. Although providing high accuracy, it demands extensive calculations and powerful computing hardware.

The classification provided is justified by enabling optimal method selection based on specific practical fractal analysis tasks, available computational resources, and accuracy requirements. Simple methods are preferable for quick assessments or limited computational capacities, while complex methods suit specialized scientific research requiring high precision. Using overly complex methods may lead to substantial time and resource expenses. Additionally, consideration should be given to the suitability of methods for discrete or continuous data.

Methods for calculating fractal dimensions should also be classified according to the **dimensionality of the space** containing the analyzed object. This criterion holds both theoretical and practical importance, as different methods vary in efficiency depending on the geometric characteristics of fractals and data acquisition capabilities for their analysis and comparison.

Methods for Analyzing Objects in One-Dimensional Space (Fig. 2) (Zero-Dimensional and One-Dimensional):

– Line-Counting Method. Divides linear structures into intervals. This method evaluates geometric characteristics of curves, fractal dimensions of coastlines, and curvilinear boundaries of fractals.

– Covering Method. Determines the minimal number of circles required to cover a fractal object. Frequently used in biology for analyzing vascular structures.

Both methods are straightforward and suitable for analyzing simple experimental data. However, their main drawbacks are limited applicability and difficulty in generalization.

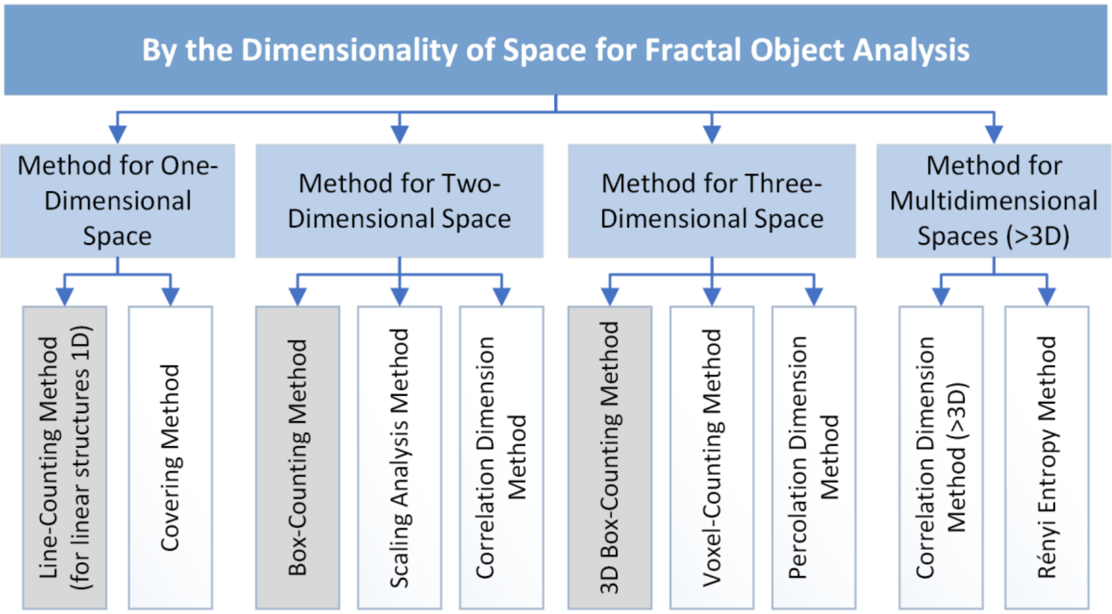


Fig. 2. Classification of fractal calculation methods by spatial dimensionality

Methods for Analyzing Objects in Two-Dimensional Space (Fig. 2):

– Box-Counting Method. Popular due to simplicity and capability of calculating dimensions for planar fractals of any shape. Effective for analyzing two-dimensional textures, cloud images, and topographic map characteristics.

– Scaling Analysis Method. Identifies self-similarity properties of fractal structures based on scaling changes in analyzed images. Applicable in geophysics, astronomy, and visualization of X-ray images.

– Correlation Dimension Method. Calculates fractal dimension through statistical analysis of distances between individual points. Widely used in ecology, engineering tasks, and particle clustering analysis.

Methods for Analyzing Objects in Three-Dimensional Space (Fig. 2) (Volumetric Fractals):

– 3D Box-Counting Method. A generalization of the grid method to three-dimensional spaces. Efficient for porous media, biological tissue structures, and cosmic object distributions.

– Voxel-Counting Method. Calculates fractal dimensions of objects in 3D models and tomographic data analysis. Beneficial for medical and materials science fields but requires specialized measurement equipment.

– Percolation Dimension Method: Differs from counting methods, focusing on analyzing geometric structures of connected clusters in porous media. Crucial for hydrodynamic research and underground reservoir analysis, yet requires specialized instrumentation.

Methods for Analyzing Objects in Multidimensional Space (Fig. 2) (>3D):

– Generalization Correlation Dimension Method. Rarely used method, applicable in chaos theory and dynamic systems analysis. Its effectiveness needs further study. Potentially useful in physical process research, economic analysis, and neuroscience, though difficult to generalize geometrically.

– Rényi Entropy Method. Utilized in statistics to determine data irregularity levels with complex parameterizations. Its effective application requires advanced computational resources and multidimensional optimization equipment.

The classification of methods based on the dimensionality of the space demonstrates how the effective choice of method significantly affects calculation complexity and computational time. Certain methods perform well only within defined-dimensional spaces and cannot be geometrically generalized. Nonetheless, this classification aids in the effective selection of methods tailored to specific practical tasks and experimental data availability.

A primary goal of classifying known fractal calculation methods is determining their **universality and standardization** for application in engineering research involving various technical objects and processes. For most practical tasks, the interrelation between calculated fractal characteristics and technological quality indicators is more critical than the accuracy of fractal dimension calculations. Thus, universality and standardization criteria are crucial, as specialized methods typically address only specific industry-related tasks.

Typological analysis of existing fractal calculation methods identified *universal and standardized methods* (Fig. 3), applicable across different spatial dimensions (1D, 2D, 3D, N-dimensional) and commonly accepted in various science and technology fields:

– Box-Counting Method. The most universal method applicable to 1D, 2D, 3D, and N-dimensional spaces. Commonly used in physics, biology, computer graphics, geophysics, and image processing. Limitations include difficulty analyzing internal geometric structures and potential errors from incorrect scale parameter selection.

– Correlation Dimension Method. Applied in dynamic systems, neuroscience, and economics. It relies on statistical correlations between fractal points and theoretically works in arbitrary-dimensional spaces but requires specialized algorithms and significant computational resources.

– Mass-Radius Method. Determines fractal dimension based on changes in point quantities depending on the radius of covering circles or spheres. Applicable in spaces of varying dimensionality but dependent on optimal sphere center selection and result averaging.

– Rényi Entropy Method. Evaluates the complexity of fractal structures in spaces of arbitrary dimensions. Requires powerful computing equipment and sophisticated multidimensional optimization algorithms.

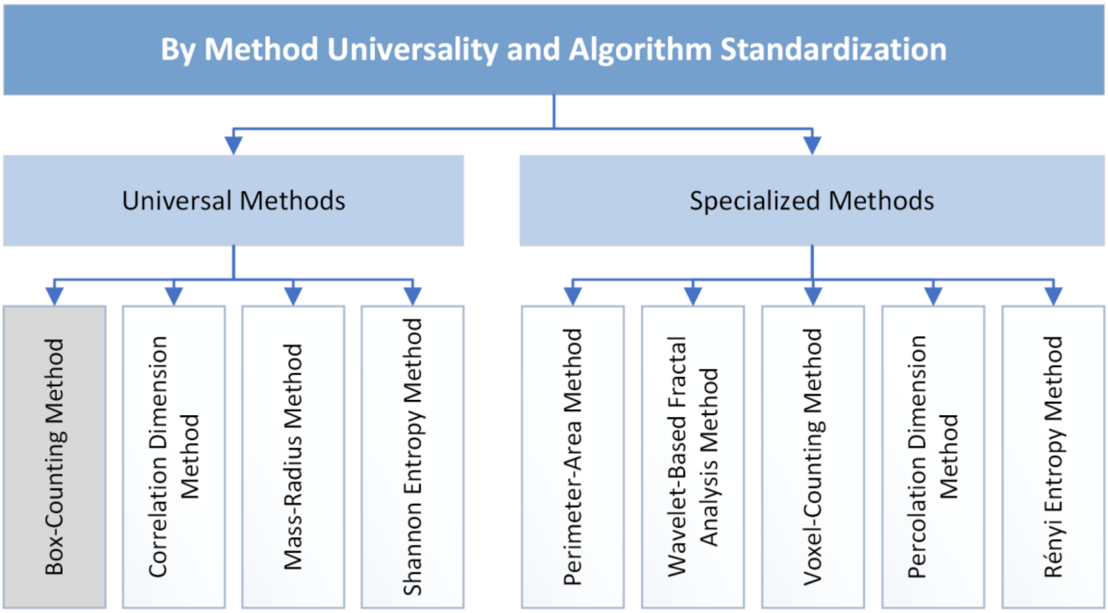


Fig. 3. Classification of fractal calculation methods by universality and algorithm standardization

Specialized methods limited by dimensionality and application fields (Fig. 3) are effective only in specific spaces (e.g., solely 2D or 3D) and used exclusively for solving specialized fractal analysis tasks:

– Perimeter-Area Method. Limited to 1D and 2D objects, unsuitable for multidimensional fractal analysis.

– Voxel-Counting Method. Narrowly specialized for 3D objects (medical, geophysical, tomography). Cannot be generalized without substantial modifications.

– Wavelet-Based Fractal Analysis Method. Mainly used for signals and images (1D, 2D) and not generalizable to higher-dimensional spaces, restricted primarily to porous cluster media analysis.

This type of classification makes it possible to clearly distinguish between general methods that produce the same results regardless of the dimensionality of the space in which the calculations are performed and to outline the range of areas of their effective use.

2. Analysis of Mathematical Foundations of Popular Computational Algorithms for Calculating Fractal Parameters of Multifractal and Quasi-fractal Objects

The conducted typological classification has identified several promising methods for addressing practical fractal analysis tasks related to automotive objects and processes. However, comparing the algorithmic complexity of the selected methods is necessary to choose the optimal approach in terms of universality and computational simplicity of fractal calculations.

Correlation Dimension Method involves utilizing a correlation function of the form:

$$C(\alpha) = \frac{1}{N^2} \sum_{i,j} \varphi\left(\alpha - \left|x_i - x_j\right|\right), \tag{1}$$

where φ is the Heaviside function, N is the total number of points in the set, and $\left|x_i - x_j\right|$ represents the Euclidean distance between points.

The correlation dimension calculates how the number of point pairs separated by a distance smaller than a scale parameter α changes. The more chaotic or ordered the points, the sharper the scale-dependent growth of this number of pairs. Fractal dimension is determined by:

$$D = \lim_{\alpha \to 0} \frac{\ln C(\alpha)}{\ln \alpha}. \tag{2}$$

Despite its specific scope of practical applications, the Correlation Dimension Method has significant drawbacks: it is effective only with zero-dimensional sets (points), sensitive to sample sizes and various scaling noises, and requires extensive computations.

Perimeter-Area Method algorithmically computes the ratio of the perimeter to the area of specified objects at different scaling coefficients. Fractal dimension is defined as:

$$D \approx 2 \frac{\ln P}{\ln S}, \tag{3}$$

where P represents the perimeter, and S is the area of the object set.

Disadvantages include its limited applicability to two-dimensional linear transport-related objects. The method becomes complicated when working with curved contours such as urban transport route structures or determining transport flow optimization based on safety parameters. Additionally, it is highly sensitive to scaling noises.

Mass Dimension Method evaluates how the quantity of points (mass) within a circle, sphere of radius α, or cube with side α varies depending on α itself. For fractal structures, a power-law relationship holds:

$$M(\alpha) \sim \alpha^D. \tag{4}$$

Thus,

$$D = \lim_{\alpha \to 0} \frac{\ln M(\alpha)}{\ln \alpha}, \tag{5}$$

where $M(\alpha)$ is the number of points within the sphere or cube of size α.

Advantages of this method include ease of implementation and applicability to objects without clearly defined boundaries (e.g., particle distribution in automotive powder coating or fuel spray efficiency analysis). However, it cannot be used effectively for analyzing objects with varying fractal scales.

Wavelet Transform Method employs wavelet transformation to decompose signals or images into components at various scales (frequencies). Fractal dimension is estimated by the energy decay of wavelet coefficients as scale changes:

$$E(\alpha) \sim \alpha^{-D}, \tag{6}$$

where $E(\alpha)=\frac{1}{N}\sum_{i}\left|W_{\alpha,i}\right|^{2}$, $E(\alpha)$ is the average energy of wavelet coefficients at scale α, $W_{\alpha,i}$ represents wavelet coefficients, and N is the total number of points.

Main disadvantages include dependency on the chosen wavelet function (e.g., Daubechies, Haar, Morlet), limited scale resolution, sensitivity to scaling noise, poor handling of boundary effects, and high computational complexity.

Minimal Covering Method. This method represents a classical approach to determining the fractal dimension of objects. It is employed for analyzing chaotic point distributions, linear structures, and geometric figures in 2D and 3D spaces. The fundamental principle involves covering a fractal object with minimal-sized elements (such as squares, cubes, etc.). The total number of covering elements, $N(\alpha)$, is computed using a movable grid, followed by the calculation of its dependence on α. The fractal dimension is subsequently determined using a power-law relationship:

$$N(\alpha) \sim \alpha^{-D}, \tag{7}$$

$$D=\lim_{x\to\alpha}\frac{\ln N(\alpha)}{\ln(\frac{1}{\alpha})}. \tag{8}$$

Drawbacks include sensitivity to the chosen minimal coverage scale range, increased computational complexity, uneven grid coverage, and difficulties applying minimal grids above two dimensions, significantly complicating the algorithm.

Most algorithmic approaches mentioned primarily focus on calculating fractal geometric characteristics of self-similar two-dimensional objects. They are ineffective when analyzing multifractal and quasi-fractal structures, requiring segmentation of binary images and determination of topological and fractal dimensions of segments or entire models. Extending these methods to three-dimensional objects is challenging due to difficulties accurately identifying geometric parameters such as distances, perimeters, areas, and occupied versus unoccupied cells within chaotic fractal structures. Consequently, linking technical and technological properties of analyzed objects to fractal characteristics of constructed models in practical applications remains a complex problem.

Special attention should be given to the Box-counting Method, which is similar algorithmically to the Minimal Covering Method and particularly effective for a wide range of practical tasks. Its essence involves overlaying a uniform grid composed of cells of a certain size onto a model and counting the number of cells containing the fractal body of the object. This method is an optimal approach for determining the fractal dimension of objects in practical problems across various fields. It facilitates the analysis of geometric characteristics of point sets, linear structures, planar, and spatial figures in two- and three-dimensional spaces, and can be generalized to spaces of arbitrary dimensions.

The method's two primary disadvantages include, on one hand, the correct selection of the minimal necessary scale coverage when analyzing fractal characteristics of a specific object and, on the other hand, the challenges associated with overlaying and visualizing hyper-grids for counting cells occupied by fractals in multidimensional spaces.

As mentioned above, the main idea is to cover the fractal object with elements of a uniform grid of required size α. The scale coverage sizes can initially be minimal, aligned directly with the objectives of a specific practical task. After applying the uniform grid, the total coverage number of elements $N(\alpha)$ in the image is determined. The dependency $N(\alpha)$ on α is calculated using a power-law relationship:

$$N(\alpha) \sim \alpha^{D}, \tag{9}$$

and the fractal dimension is computed as:

$$D \approx \frac{\ln N(\alpha)}{\ln \alpha}. \tag{10}$$

Applying the Box-counting method involves discretely covering the studied object’s image with a set of cells arranged in a grid with a particular scale factor. For effective identification and precise analysis of geometric characteristics of such fractal models, grid element sizes are optimally chosen relative to the object and may approach the pixel size for two-dimensional raster images.

Works [10-14] describe developed approaches and methods for effectively applying the Box-counting method as a universal tool for fractal analysis of technological and safety characteristics of objects across various contemporary production and technological fields.

For instance, study [10] proposes a method of quantitative fractal diagnostics of diesel engine fuel injectors (Fig. 4). Experimental studies of fuel spray quality using repaired injectors have established a direct relationship between the fractal dimension of spray patterns and wear levels of precision pairs in fuel equipment. The experimental results indicate that the fractal dimension of the spray pattern can effectively diagnose injector readiness for engine operation or identify the necessity for repairs or replacement.

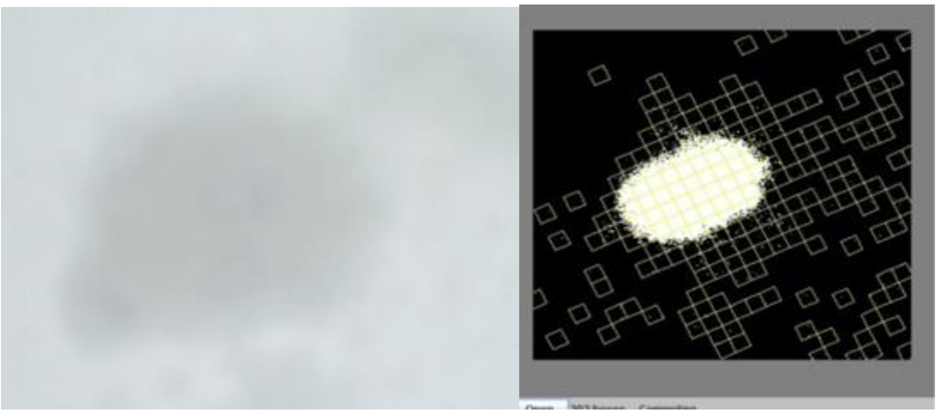


Fig. 4. Fractal assessment of fuel spray quality in internal combustion engine injectors

Research [11] presents a method of quantitative fractal assessment of powder coating quality, facilitating effective management of powder charging processes considering different particle dispersions and moisture levels. Computer implementation of quality evaluation and management processes for powder paint application was executed (Fig. 5). The studies suggest that significant optimization of tribostatic powder coating processes is possible in small enterprises within the mechanical engineering or automotive industries.

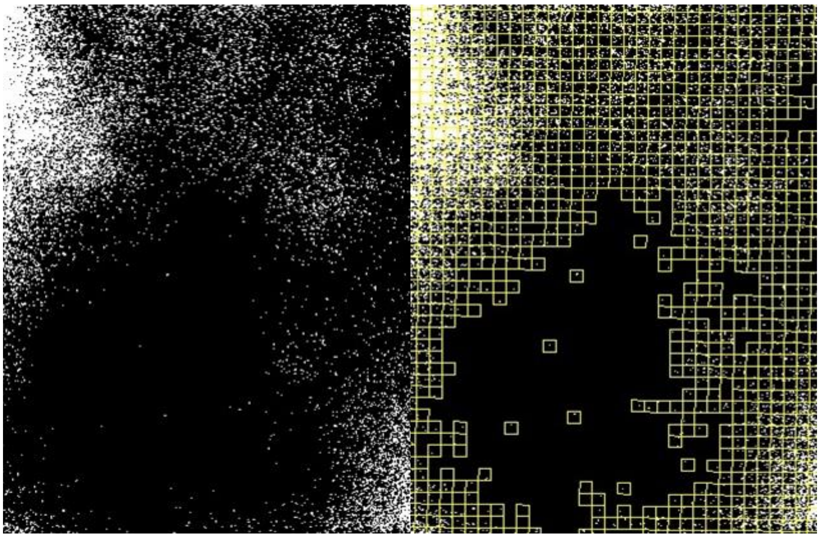
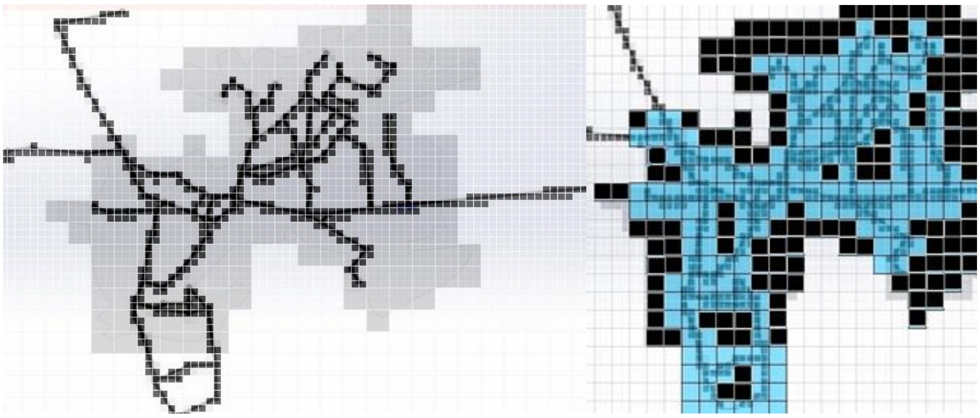


Fig. 5. Fractal analysis of tribostatic powder coating

Study [12] investigates the influence of geometric parameters, including fractal ones, of discrete urban transportation network models on passenger transportation quality and improvement pathways for the structural components of the transportation network. Methods for identifying route diagrams (Fig. 6-a) were developed, highlighting essential geometric (fractal) elements of the discrete urban transportation network model significantly impacting passenger transportation quality. Algorithms and methods for computer calculations of fractal characteristics of identified images are presented, facilitating effective adjustments of the technical and technological properties of urban transport infrastructure.



a)b)

Fig. 6. Fractal analysis of transportation route network quality and route overlap indicators

Research [13] addresses urban passenger transportation features and the impact of fractal characteristics of transport systems on evaluating route overlap indicators (Fig. 6-b). A fractal method was proposed to assess route overlap in specific zones and optimize transport vehicle operation on routes during various daily periods. Research outcomes facilitated effective route modifications, timely adjustments to carriers' schedules, normative execution times for trips, driver labor organization systems, and combined operational modes for urban passenger transport.

However, applying the universal Box-counting method becomes problematic when analyzing fractal models in three-dimensional or especially multidimensional spaces due to the absence of computational tools for calculating geometric characteristics of specific internal regions and the model overall.

Study [14] developed an innovative method for stacking box-shaped cargo in automotive transport, significantly reducing cargo unit blocking costs by designing specialized loading schemes. The study employed fractal mathematics to assess the quality of blocking cargo with hazardous materials, considerably lowering securing costs and optimizing safety criteria for hazardous materials transportation. The primary issue was identifying internal geometric characteristics of the fractal model (Fig. 7), raising questions about linking fractality across individual layers and determining each layer's influence on technological and safety characteristics of the stacked cargo.

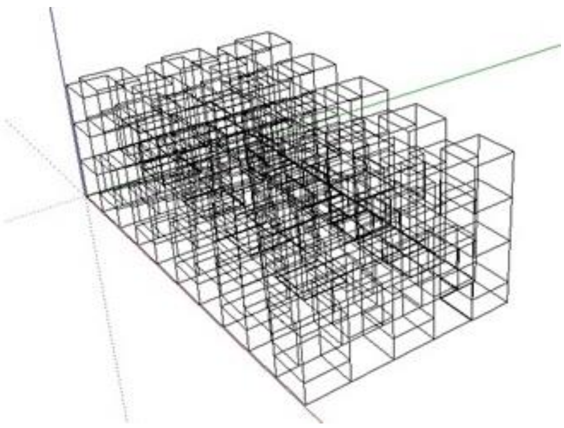


Fig. 7. Three-dimensional model for stacking box-shaped cargo on freight vehicles

Authors addressed this specific problem by developing a methodology for evaluating and comparing fractal parameters of separate object projections (Fig. 8). However, this approach may not be efficient or applicable for similar tasks involving the determination of fractal characteristics in multidimensional object models across other technical fields.

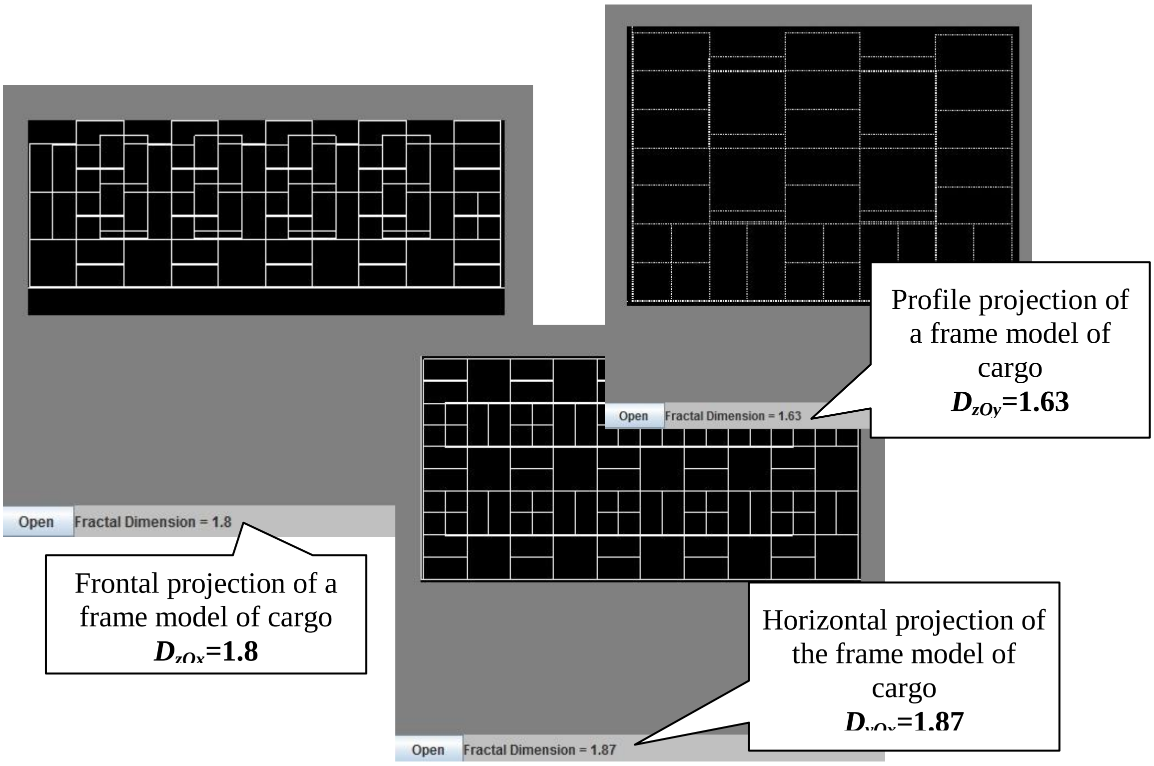


Fig. 8. Methodology for fractal quality assessment of cargo stacking based on separate projections

3. Relationship Between Fractal Parameters of Quasi-Fractal and Multifractal Computer Models and Technological, Technical, and Economic Characteristics of Automotive Industry Objects and Processes

The authors' experience in developing fractal analysis methods for various technical problems has allowed establishing relationships between fractal geometric characteristics and technological qualities of objects and processes in the automotive industry, urban transport networks, logistics operations, infrastructure optimization, transport flow efficiency, and road safety.

Fractal dimension indicators of urban and intercity roads determine their branching degree and spatial occupancy. Networks with low fractal dimensions ($D < 1.5$) are characterized by simple, linear structures with limited routes and low throughput. Networks with high fractal dimensions ($D > 1.7$) have more complex interconnections, a greater number of alternative routes, and enhanced transport accessibility. Thus, applying fractal dimension calculation methods in urban road modeling can assess how effectively the future network will adapt to the urban environment, identify bottlenecks, and predict congestion levels. Road network optimization based on fractal characteristics can reduce average travel time and vehicle energy consumption.

Correct calculation methods for fractal parameters of road surfaces allow determining their roughness, directly influencing tire-road adhesion and driving safety. Road surfaces with excessively low fractal dimensions ($D < 2.3$) result in poor tire-road adhesion, particularly on wet and snowy roads. Conversely, surfaces with excessively high fractal dimensions ($D > 2.6$) lead to rapid tire wear and increased vehicle energy consumption. Such fractal analysis facilitates determining the optimal road texture for maximum safety and minimal vehicle wear.

Urban traffic flows also exhibit fractal structures, formed through complex interactions between vehicles, traffic lights, and road infrastructure. A low fractal dimension of traffic flow ($D < 1.2$) indicates regular traffic, low transport density, and minimal congestion influence. Conversely, excessively high fractality ($D > 1.5$) reflects chaotic traffic conditions, significant variability in flow density, potentially causing accidents and congestion. Fractal analysis thus provides a numerical measure for predicting traffic intensity and adaptive traffic light adjustments to mitigate congestion.

Fractal analysis methods can effectively optimize urban development parameters and automotive transport network accessibility. Fractality serves as a qualitative measure of how efficiently urban structures organize roads, residential areas, and transport hubs. Compact cities with fractal dimensions around $D \approx 1.5$ offer improved transport accessibility and reduce average travel distances. In contrast, highly branched cities with high fractal dimensions ($D > 1.8$) feature complex transport structures and longer travel times. Such calculations allow for optimal forecasting of the impact urban planning changes have on transportation efficiency.

Correctly calculated fractal parameters can be effectively utilized in logistics for managing urban goods distribution. Routes with low fractal dimensions (D < 1.3) display centralized patterns, vulnerable to congestion. Conversely, high fractality patterns (D > 1.5) offer adaptive structures, improving resilience to delays and disruptions. Fractal analysis identifies weak points in freight flows and facilitates more flexible logistics schemes.

Additionally, fractal indicators of warehouse systems influence logistical route efficiency, directly impacting goods distribution principles. Warehouses with low fractal dimensions (D < 1.2) have simple structures but limited adaptability to demand fluctuations. Warehouses with higher fractality (D > 1.4) provide better flexibility and faster cargo handling. Here, fractal analysis improves inventory management, minimizing delays and optimizing costs.

Many practical tasks in automotive transportation relate to optimizing safety characteristics. In this context, fractal calculation methods enable predicting hazardous road sections, improving road junction designs to reduce accident probabilities, developing adaptive traffic management systems through chaotic parameter calculations, and reducing network congestion by optimizing transport infrastructure.

DISCUSSION OF RESEARCH RESULTS

The research conducted within this work, particularly regarding the applicability of analyzed methods for calculating fractal dimensions, their classification according to defining characteristics, and analysis of their algorithmic foundations, leads to the conclusion that the Box-Counting Method is the most effective and versatile. This method clearly visualizes the geometric complexity of object models and their self-organization. Its algorithm is sufficiently simple, enabling quantitative assessment of hierarchical organization and structural chaos levels of analyzed objects. This directly influences the accurate determination of interrelations between fractal parameters and technological properties of real physical phenomena and processes.

If the proposed concept of universality for the Box-Counting Method is accepted, and further theoretical, methodological, and algorithmic components are expanded and deepened, the primary remaining challenge is the absence of a visualization principle for the internal structures of studied objects in spaces of dimensionality greater than two. Reducing the procedure for calculating the fractal dimension of objects, even in three-dimensional space, to analyzing characteristics of individual projections often fails to yield the desired results. Therefore, an innovative approach to calculating fractal parameters has been proposed, based on continuously reducing the dimensionality of the space in which the models are located.

Practically, this procedure resembles progressively layering a multidimensional object into separate layers of varying dimensionalities, determining the fractal parameters of each layer, and consolidating geometric data into a unified informational framework. This form of fractal scanning allows for adjustments to individual layers, elements, or nodes of the model, thereby facilitating further enhancement of its technological and technical characteristics.

Initial computational experiments have produced promising results, demonstrating broad potential for generalization of this methodology to spaces of arbitrary dimensionality.

CONCLUSIONS

1. All research objectives aimed at achieving the goal of this study have been fulfilled. A typological classification of fractal dimension calculation methods was conducted, positioning them as instrumental tools for addressing practical tasks across various scientific and technical fields. This classification outlined critical geometric criteria for the optimal selection of universal methods when solving design tasks within automotive and transportation technologies.

2. Computational algorithms for determining fractal parameters of objects were studied, particularly concerning their generalization potential and applicability to spaces of arbitrary dimensions. The strengths and limitations of these methods were identified, and ideas for developing an innovative fractal parameter calculation method based on the dimensionality reduction paradigm were proposed.

3. Key elements reflecting the interrelation and mutual influence between fractal parameters of quasi-fractal and multifractal computer models and technological, technical, and economic characteristics of automotive objects and processes were identified. The impact of each studied parameter on improving transport object design processes was analyzed.

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Пустюльга С.І., Самчук В.П., Головачук І.П., Приступа О.В., Лелик Я.Р. **Аналіз методів розрахунку фрактальної розмірності як інструменту удосконалення експлуатаційних та безпекових характеристик об'єктів автомобільного транспорту**

Робота присвячена розробці концепції ефективного використання методів аналізу та розрахунку фрактальної розмірності моделей у дво-, тривимірному та багатовимірних просторах для вирішення практичних завдань удосконалення якісних характеристик об'єктів автомобільного транспорту, транспортної інфраструктури та логістики. Визначено найбільш універсальний та обчислювально ефективний метод розрахунку фрактальних параметрів, який відкриває нові можливості для оптимізації процесів моделювання реальних систем і суттєво підвищує якість їх аналізу та прогнозування.

Сучасні прикладні задачі, у тому числі і в галузі транспортних технологій, активно рухаються в бік дослідження процесів у багатовимірних просторах, які пов'язані із вивченням динаміки поведінки хаотичних систем. Пошук універсальних методів і алгоритмів розрахунку фрактальних параметрів моделей багатопараметричних об'єктів у просторах довільної розмірності, які змінюють свої властивості, наприклад в часі, вимагає додаткових досліджень для їх узагальнення.

Аналіз літературних джерел показав, що отримані у них результати включають ряд невирішених задач фрактального аналізу. Вони стали основою для формулювання завдань роботи, серед яких: класифікація методів розрахунку фрактальної розмірності, як інструменту розв'язання практичних задач у різних галузях науки та техніки; дослідження обчислювальних алгоритмів підрахунку фрактальних параметрів об'єктів, з точки зору можливості їх узагальнення та використання, для просторів довільного числа вимірів; визначення взаємовпливу та взаємозв'язку фрактальних параметрів квазіфрактальних і мультифрактальних комп'ютерних моделей із технологічними, технічними та економічними характеристиками об'єктів та процесів автомобільної галузі. На цій основі окреслено шляхи розробки ефективного, в сенсі практичних розрахунків та аналізу фрактальних об'єктів, методу, основними характеристиками якого є універсальність та стандартизованість.

Виконані у роботі дослідження щодо можливостей використання проаналізованих методів розрахунку фрактальної розмірності, їх класифікація за визначальними ознаками, аналіз їх алгоритмічної основи, дозволили зробити висновок, що найбільш ефективним і універсальним є - Box-Counting Method. Він відкриває можливості для максимально зрозумілої візуалізації геометричної складності моделей та їх самоорганізацію. Алгоритм методу достатньо простий і дозволяє кількісно оцінити рівень ієрархічної впорядкованості та хаотичності структури досліджуваних об'єктів, який прямо впливає на коректне визначення взаємозв'язків між параметрами фрактальності та технологічними властивостями реальних фізичних явищ та процесів.

Запропоновано концепцію щодо удосконалення найбільш універсального методу коробок, окреслено шляхи розширення та поглиблення його теоретичної, методологічної, алгоритмічної складових. Висунуто ідею розрахунку фрактальних параметрів моделей об'єктів у просторах довільного числа вимірів шляхом неперервного пониження розмірності простору, в якому вони знаходяться. Така процедура схожа, в практичному сенсі, із поступовим розшаруванням багатовимірного об'єкту на окремі шари з різною розмірністю, визначення параметрів фрактальності таких шарів і зведення геометричних даних у єдину інформаційну основу. Таке своєрідне фрактальне сканування дозволить коригувати окремі шари, елементи чи вузли моделі для подальшого удосконалення її технологічних та технічних властивостей.

Ключові слова: фрактальна розмірність, технологічні та експлуатаційні характеристики об'єктів транспорту, багатовимірний простір, концепція взаємозв'язку геометричних параметрів та якісних властивостей об'єктів транспортної інфраструктури.

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