INVESTIGATION OF DYNAMICS OF THE MANIPULATOR WITH ROTATIONAL EXCITATION

INTRODUCTION

Dynamics of the manipulator with rotational excitation is investigated in this paper. First the model with excitation of limited power is described in detail and dynamics for typical parameters of the system is investigated. Then the manipulator with excitation of unlimited power is described and investigated. Transient and steady state motions for typical parameters of the system are presented.

Results are applied in the process of design of manipulators with rotational excitation. Robot performing stepping motion is proposed in [1]. Robot with impact interactions is investigated in [2]. System with two sided impacts is analyzed in [3]. Pipe robot with self-stopping mechanism is investigated in [4]. Soft impacts are analyzed in [5]. Vibrating systems with impacts are investigated in [6]. Vibrations in transmissions are analyzed in [7]. Synchronization of mechanical systems is investigated in [8]. Precise robots are described in [9]. Industrial robots are analyzed in [10]. New mechanisms in contemporary robot engineering are presented in [11]. Mechanics of vibrating systems is investigated in [12].

In this paper displacements, velocities, angle, angular velocity, difference of displacements and difference of velocities as functions of time are investigated. Graphical relationships for different values of stiffness are presented. They enable us to understand the influence of the value of stiffness on the dynamic behavior of the manipulator.

MODEL OF THE MANIPULATOR WITH ROTATIONAL EXCITATION

Manipulator with rotational excitation is shown in Fig. 1.

The following notation is introduced:

\[ AB = r. \] (1)
Figure 1. Model of the system: \(XOY\) – the immovable coordinate system; 1 – mass \(m_1\) of the disbalance of the rotor rotating about the point \(A\); 2 – impacting mass \(m_2\) of the vibrator; 3 – immovable straight linear supports, which ensure the direction of the moving mass \(m_0\).

The coordinates of the main points determining the investigated system are:
\[ O(0,0); O_1(x_0-r,0), A(x_1,0); B(x_r, y_r). \]  
(2)

Location of the exciting mass of the exciter of vibrations is expressed as:
\[ x_i = x_0 + r \cos \varphi, \]
\[ y_i = -r \sin \varphi. \]  
(3)  
(4)

Velocities are determined as:
\[ \dot{x}_i = \dot{x}_0 - r \dot{\varphi} \sin \varphi, \]
\[ \dot{y}_i = -r \dot{\varphi} \cos \varphi. \]  
(5)  
(6)

Kinetic energy has the following form:
\[ T = \frac{m_1}{2} \left[ \dot{x}_0^2 + \left( r \dot{\varphi} \right)^2 - 2 \dot{x}_0 \dot{\varphi} \sin \varphi \right] + \frac{m_2}{2} \dot{x}_1^2 + \frac{m_3}{2} \dot{x}_2^2. \]  
(7)

Derivatives of the kinetic energy have the following forms:
\[ \left( T' \right)_i = (m_1 + m_0) \ddot{x}_0 - m_1 r (\ddot{\varphi} \sin \varphi + \dot{\varphi}^2 \cos \varphi), \]  
\[ T'_{\varphi} = m_1 r (\ddot{\varphi} - \dot{x}_0 \sin \varphi), \]  
\[ \left( T'' \right)_i = T''_{\varphi} = m_1 r (r \ddot{\varphi} - \dot{x}_0 \sin \varphi). \]  
(8)  
(9)  
(10)

Potential energy has the following form:
\[ \Pi = \frac{1}{2} C (x_2 - x_0 - l)^2. \]  
(11)

Potential forces have the following forms:
\[ \Pi'_{x_2} = C(x_2 - x_0 - l), \]  
\[ \Pi'_{x_0} = C(x_0 - x_2 + l). \]  
(12)  
(13)

Dissipative function has the following form:
\[ D = \frac{1}{2} H (\ddot{x}_2 - \dot{x}_0)^2 + \frac{1}{2} H_{x_0} \dot{x}_0^2. \]  
(14)

Dissipative forces have the following forms:
\[ D'_{x_2} = H (\ddot{x}_2 - \dot{x}_0), \]  
(15)
According to the coordinate $x$, the following force is acting:
\[ Q_0 = -A - B \dot{x}_0, \]  
and according to the coordinate $\varphi$ the following moment of the force is acting:
\[ M_\varphi = C - D \ddot{\varphi}, \]
where $A$, $B$, $C$ and $D$ are constant positive quantities.

When the following condition is satisfied:
\[ x_2 - x_0 < l, \]  
dynamics of the system is described by the following equations:
\[ \ddot{x}_2 + p^2 (x_2 - x_0 - l) + h (\dot{x}_2 - \dot{x}_0) = 0, \]
\[ (1 + \mu) \ddot{x}_0 - r (\dot{\varphi} \sin \varphi + \dot{\varphi}^2 \cos \varphi) + p^2 (x_0 - x_2 + l) + h_0 \dot{x}_0 = -a - b \dot{x}_0, \]
where it is denoted:
\[ \mu = \frac{m_2}{m_0}, \quad a = \frac{A}{m_2}, \quad b = \frac{B}{m_2}, \quad c = \frac{C}{m_2}, \quad d = \frac{D}{m_2}, \quad h = \frac{H}{m_2}, \quad h_0 = \frac{H_0}{m_2}, \quad p^2 = \frac{C}{m_2}. \]

When the following condition is satisfied:
\[ x_2 - x_0 = l, \]
impact of the mass $m_2$ with the mass $m_0$ takes place in a moment of time, where the change of velocities of those masses takes place according to the theorem of quantity of motion and losses of velocity of impact, that is:
\[ \dot{x}_2^+ + \mu \dot{x}_0^+ = \dot{x}_2^- + \mu \dot{x}_0^-, \]
\[ R = \frac{\dot{x}_2^- - \dot{x}_0^-}{\dot{x}_2^+ - \dot{x}_0^+}. \]

Thus, it is obtained that:
\[ \dot{x}_2^+ = \frac{1}{1 + \mu} \left[ (1 + R) \dot{x}_2^- + (\mu - R) \dot{x}_0^- \right], \]
\[ \dot{x}_0^+ = \frac{1}{1 + \mu} \left[ (1 - \mu R) \dot{x}_2^- + \mu (1 + R) \dot{x}_0^- \right]. \]

**RESULTS OF INVESTIGATION OF DYNAMICS OF THE MANIPULATOR**

The following parameters of the investigated manipulator were assumed:
\[ l = 0.03, \quad h = 0.2, \quad \mu = 0.5, \quad r = 0.02, \quad h_0 = 0.2, \quad b = 0.2, \quad a = 0.01, \quad d = 0.2, \quad c = 0.4, \quad R = 0.7. \]

The following initial conditions were assumed:
\[ x_2 (0) = 1, \quad x_0 (0) = 0, \quad \varphi (0) = 0, \quad \dot{x}_2 (0) = 0, \quad \dot{x}_0 (0) = 0, \quad \varphi (0) = 0. \]

Results for:
\[ p = 1 \]
are presented in Fig. 2.

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a) $x_2 (t)$  
b) $\dot{x}_2 (t)$
Results for: 

\[ p = 2 \] (33)

are presented in Fig. 3.

Figure 2. Dynamics of the manipulator when \( p = 1 \)

g) \( x_1(t) - x_n(t) \)  

h) \( \dot{x}_1(t) - \dot{x}_n(t) \)
From the presented results the influence of the stiffness to the dynamic behavior of the manipulator with rotational excitation is seen.

**INVESTIGATION OF DYNAMICS FOR THE CASE OF EXCITATION OF UNLIMITED POWER**

In this case the equations of dynamics take the form presented further. When the following condition is satisfied:

\[ x_2 - x_0 < l, \]

dynamics of the system is described by the following equations:

\[
\ddot{x}_2 + p^2 (x_2 - x_0 - l) + h(\dot{x}_2 - \dot{x}_0) = 0, \\
(1 + \mu)\ddot{x}_0 + p^2 (x_0 - x_2 + l) + h(\dot{x}_0 - \dot{x}_2) + h_0\ddot{x}_0 = -a - bx_2^2 + r\omega^2 \cos \omega t,
\]

where \( \omega \) denotes the frequency of rotational excitation.

The following parameters of the investigated manipulator were assumed:

\[ \omega = 2.2, l = 0.03, \mu = 0.5, h = 0.1, b = 0.1, a = 0.01, r = 0.02, R = 0.7. \]

The following initial conditions were assumed:

\[ x_2(0) = l, \; x_0(0) = 0, \; \dot{x}_2(0) = 0, \; \dot{x}_0(0) = 0. \]

Results for:

\[ p = 1 \]

are presented in Fig. 4.

**Figure 3. Dynamics of the manipulator when \( p = 2 \)**

a) \( x_2(t) \)  

b) \( \dot{x}_2(t) \)
Results for: $p = 2$

are presented in Fig. 5.

From the presented results the influence of the stiffness to the transient processes of the manipulator with rotational excitation is seen.
RESULTS OF INVESTIGATION OF STEADY STATE MOTIONS

Results for:

\[ p = 1 \]  

are presented in Fig. 6.

\[ \dot{x}_2(t) \]  

a) \( x_2(t) \)  

b) \( \dot{x}_2(t) \)  

c) \( x_0(t) \)  

d) \( \dot{x}_0(t) \)  

e) \( x_2(t) - x_0(t) \)  

f) \( \dot{x}_2(t) - \dot{x}_0(t) \)

Figure 6. Steady state dynamics of the manipulator when \( p = 1 \)

Results for:

\[ p = 2 \]  

are presented in Fig. 7.

\[ \dot{x}_2(t) \]  

a) \( x_2(t) \)  

b) \( \dot{x}_2(t) \)  

\[ \dot{x}_2(t) \]  

a) \( x_2(t) \)  

b) \( \dot{x}_2(t) \)  

e) \( x_2(t) - x_0(t) \)  

f) \( \dot{x}_2(t) - \dot{x}_0(t) \)
CONCLUSIONS

Dynamics of the manipulator with rotational excitation is investigated. The model with excitation of limited power is described in detail and dynamics for typical parameters of the system is investigated. Displacements, velocities, angle, angular velocity, difference of displacements and difference of velocities as functions of time are investigated.

The manipulator with excitation of unlimited power is described and investigated. Transient and steady state motions for typical parameters of the system are presented. Displacements, velocities, difference of displacements and difference of velocities as functions of time are investigated.

Graphical relationships for different values of stiffness are presented. They enable us to understand the influence of the value of stiffness on the dynamic behavior of the manipulator.

Results are applied in the process of design of manipulators with rotational excitation.

REFERENCES

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Investigation of dynamics of the manipulator with rotational excitation.

Dynamics of the manipulator with rotational excitation is investigated in this paper. First the model with excitation of limited power is described in detail. The main parts of the model of the system are the mass of the disbalance of the rotating rotor, the impacting mass of the vibrator and the immovable straight linear supports, which ensure the direction of the moving mass. The system has three degrees of freedom, and its dynamics is described by the three differential equations of the second order.

Then investigation of dynamics for typical parameters of the system is performed. Displacements, velocities, angle, angular velocity, difference of displacements and difference of velocities as functions of time are investigated.

Also, the manipulator with excitation of unlimited power is described. The system has two degrees of freedom, and its dynamics is described by the two differential equations of the second order.

Then investigation of dynamics of the manipulator with excitation of unlimited power for typical parameters of the system is performed. Transient as well as steady state motions for typical parameters of the system are presented. Displacements, velocities, difference of displacements and difference of velocities as functions of time are investigated.

Graphical relationships for different values of stiffness are presented. They enable us to understand the influence of the value of stiffness on the dynamic behavior of the manipulator.

Results are applied in the process of design of manipulators with rotational excitation.

KEYWORDS: ROTATIONAL EXCITATION, MANIPULATORS, ROBOTS, GRAPHICAL RELATIONSHIPS, NONLINEAR BEHAVIOR.